STRENGTH OF MATERIALS

In Statics, we study the behaviour of rigid body, but in practice, no material is rigid and all materials deform. Study of such materials is known as mechanics of deformable bodies (or) Strength of materials.
(or)
The resistance by which the material of a body opposes the deformation is called Strength of materials.

Properties of materials

1. Rigid Material
It does not undergo any deformation when subjected to an external loading (Glass & Cast iron)

Elastic Material
It undergoes a deformation when subjected to an external loading such that the deformation disappears on the removal of the loading (Rubber)

Plastic Material
It undergoes a continuous deformation during the period of loading and the deformation is permanent. It does not regain its original dimensions on the removal of the loading (Aluminium)

Ductile Material
It undergoes large amount of deformation without rupture. It must possess a high degree of plasticity & strength (large dia into a thin wire) (Mild steel)

Brittle material
They exhibit negligible amount of deformation, over that it is fracture. It is lack of ductility & Rupture with little or No plastic deformation (Glass, Cast iron, Concrete...)

Malleability
Materials ability to be hammered out into thin sheets (lead, Slate)

2. Strength
The power of resisting loads. This is the maximum stress that a material can take. equal to Max load/Area.

Stiffness
The power of resisting deformation/deflection. The minimum strain that a material can have. equal to span/deflection

Hardness
It is the ability of material to resist against surface rupture/Abrasion. (Diamond has high hardness, whereas Chalkpiece has least hardness)

Toughness

The maximum resistance offered by a body upto the fracture.

The property which enables it to absorb energy without fracture.

Fatigue

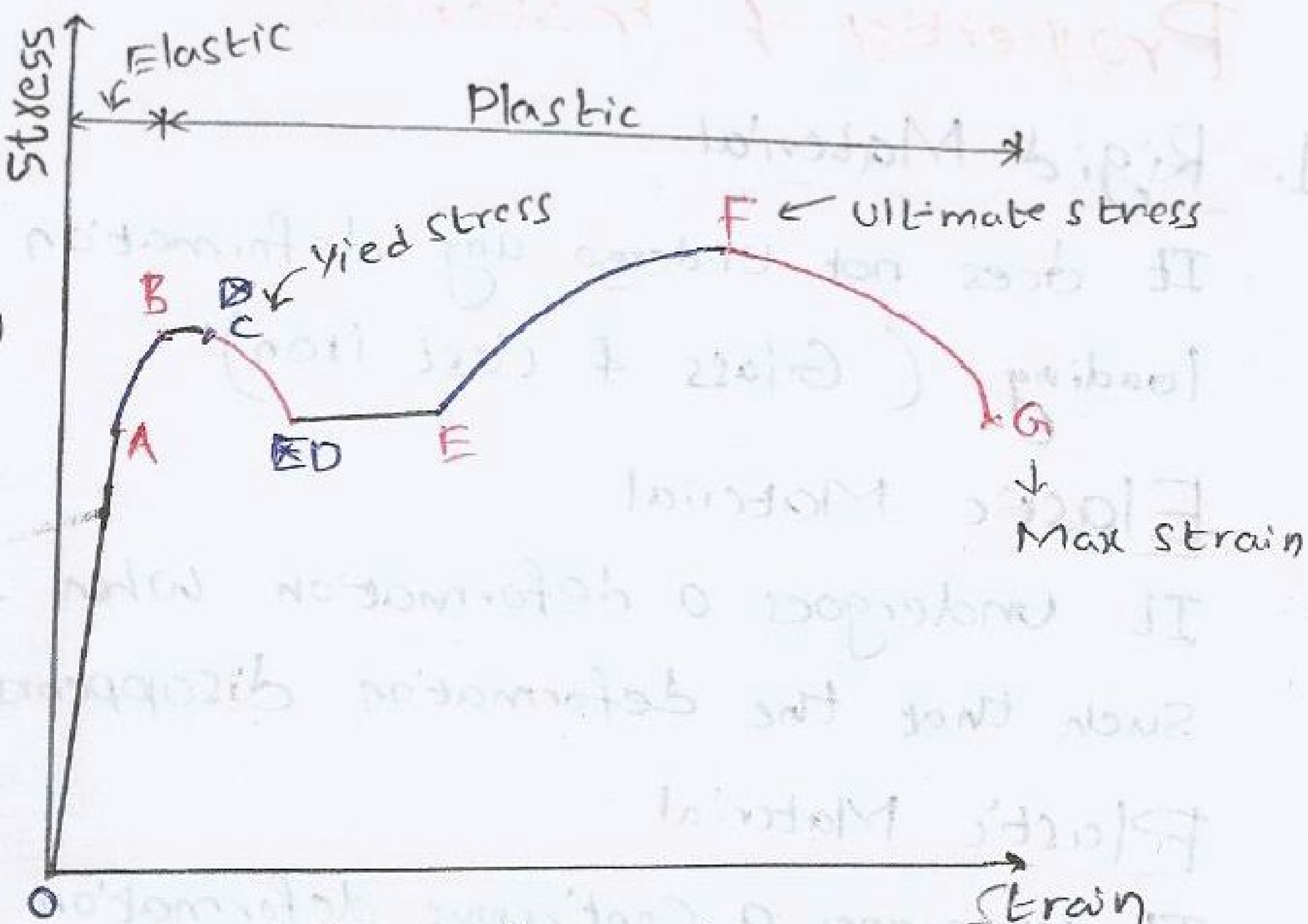
Structural damage that occurs when a material is Subjected to Cyclic loading or Repeated loading and Unloading. [And the resistance to Fatigue is called Fatigueness.]

Engineering stress-strain curve

Hooke's law

Stress is directly proportional to Strain, upto proportional Limit (i.e A)

(i.e if removal of load, it comback to original position of 100%)



Stress Strain diagram for a ductile material like mild steel.

1. **Point A** (Proportional limit)
   upto 'A' Hooke's law is obeyed.
   ∴ OA is straight line.
   also called Hooke's limit.

2. **Point B** (Elastic Limit)
   From A-B it possesses Elastic nature but it is not perfect elastic (95-98%)

3. **point C** (Upper Yielding point)
   From B-C it is in plastic state (i.e it may not come back into original position)
   From 'C' it starts yielding (i.e C/s decreases at constant/less load)

   Point D (Lower yielding point)
   From C-D the material was yielding and the resisting stress will decrease.
   From 'D' stress will not decrease but constant

   Point E (Strain hardening Region)
   At 'E' the material possess some hardening, Hence we need to increase stress to Elongate the material, up to F.

4. **point F** (Ultimate point)
   The corresponding load/stress is called 'Ultimate strength'/stress
   From 'F' the material will not resist any load and Yield continuously.

5. **point G** (Breaking point or point of fracture)
   At this point the body split into two parts.

* The fracture of ductile material is of the Cup & Cone type.

Proof stress

It is the stress at a given specified strain (i.e for proof)

→ Generally, Proof stress = stress @ 0.2% strain (i.e 0.002 strain)

Max/Ultimate Stress $= \dfrac{\text{Load at Ultimate point}}{\text{Original C/s Area before test}}$

(Yield Stress $= \dfrac{\text{Load at Yield}}{\text{original c/s}}$

Nominal Stress $= \dfrac{\text{Load at any point}}{\text{Original C/s Area before test.}}$

True (or) Actual Stress $= \dfrac{\text{Load at any Point}}{\text{C/s area at that point/load}}$

Working (or) Safe Stress

The maximum Stress to which any member is designed to resist deformation. Hence safety factor will be Consider. it is much less than Yield stress.

$\therefore$ Working stress $= \dfrac{\text{Yield stress}}{\text{Factor of Safety}}$ (Ductile materials)

$= \dfrac{\text{Ultimate stress}}{\text{Factor of Safety}}$ Brittle materials

$\because$ For brittle: Yield stress = Ultimate Stress

ie brittle materials do not possess any Yielding

Factor of Safety = 1.85 (Steel)

= 3 (Concrete)

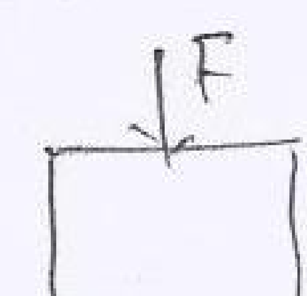
Stress

The force of resistance per unit area offered by a body against deformation is known as the Stress (or) Intensity of stress.

Direct (or) Normal stress

Stress that acts normal to the Surface (i.e load acting ⊥r to the C/s plane)

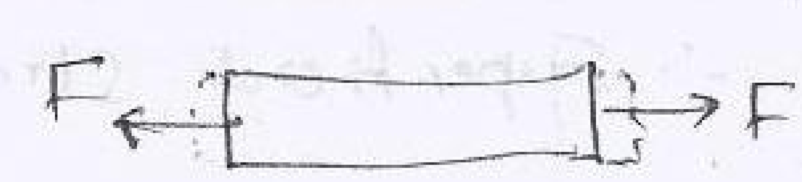
It Can be either Tensile or Compressive in nature.



$$\sigma = \frac{\text{Applied load}}{\text{original C/s area resisting the force}}$$

Tensile Stress

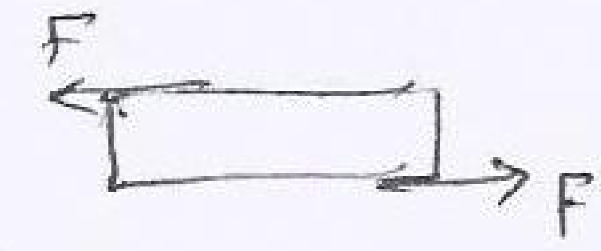
Stress that acts to lengthen an object



Compressive Stress

Stress that acts to shorten an object

Shear stress (τ)

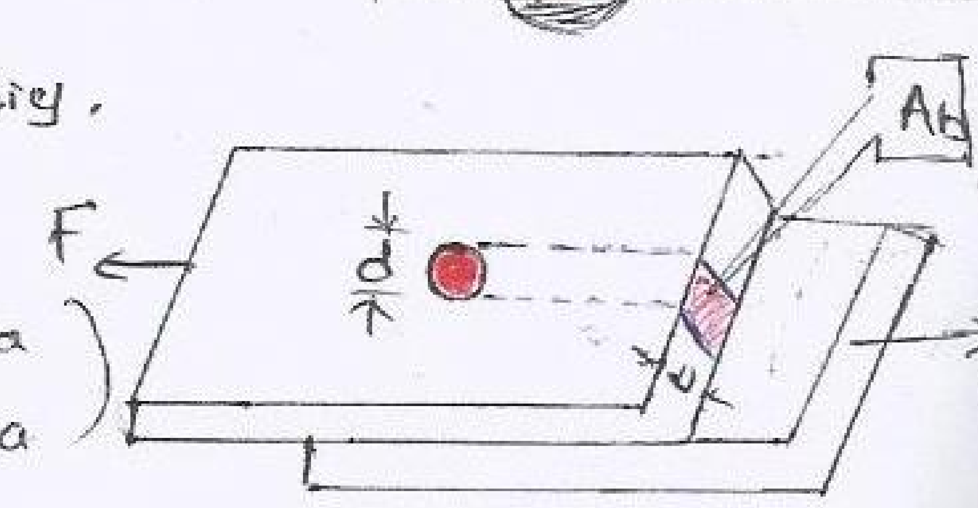


It results when a member is Subjected to a force (Called Shear force) that is parallel or tangential to the Surface.

Bearing stress (σ_b)

The contact Compressive pressure between seperated bodies, when they are Connected by Bolts, pins (or) Rivets,

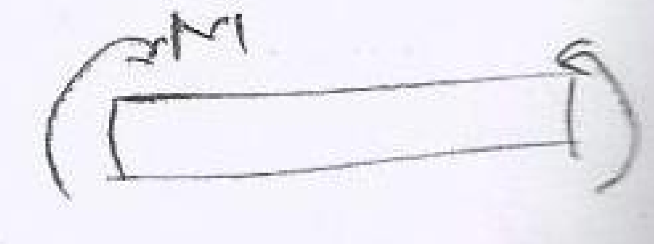
$$\sigma_b = \frac{F}{A_b} = \frac{F}{td}$$ ($\because$ A_b = Projected Contact area = Plate thickness x Bolt dia)



Bending Stress (f)

The stress developed to resist the bending moment.

It is the Combination of both Tensile and Compressive Stresses

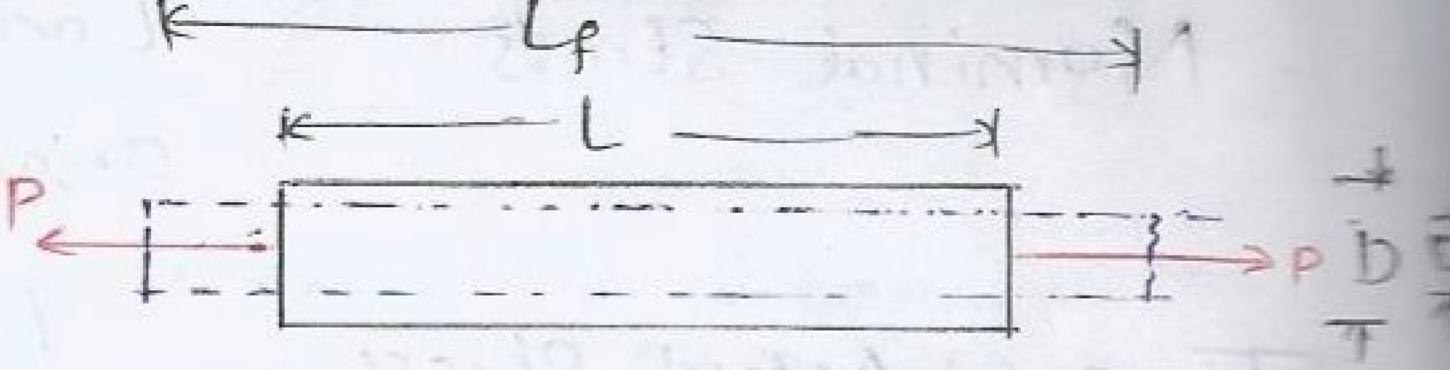


$$f = \frac{M}{Z} = \frac{\text{Bending moment}}{\text{Section modulus.}}$$

Strain (Rate of Change of dimension)

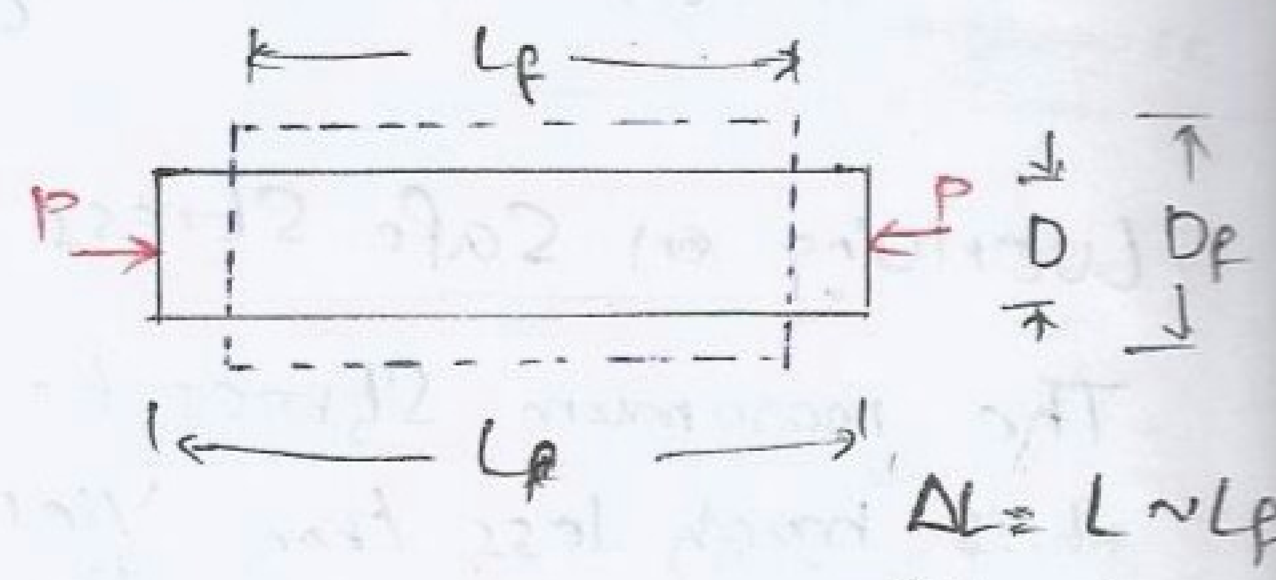
Strain is defined as an amount of deformation an object experiences compared to its original size & shape due to simple stresses.

1. Direct (or) Normal Strain (e)



Strain caused due to direct Tensile/Compressive stress/force.

$$e = \frac{\text{Change in dimension}}{\text{Original dimension}}$$



a) Longitudinal Strain (e_x)

Strain in the direction of force

$$e_x = \frac{\text{Change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

$\Delta L = L \sim L_f$

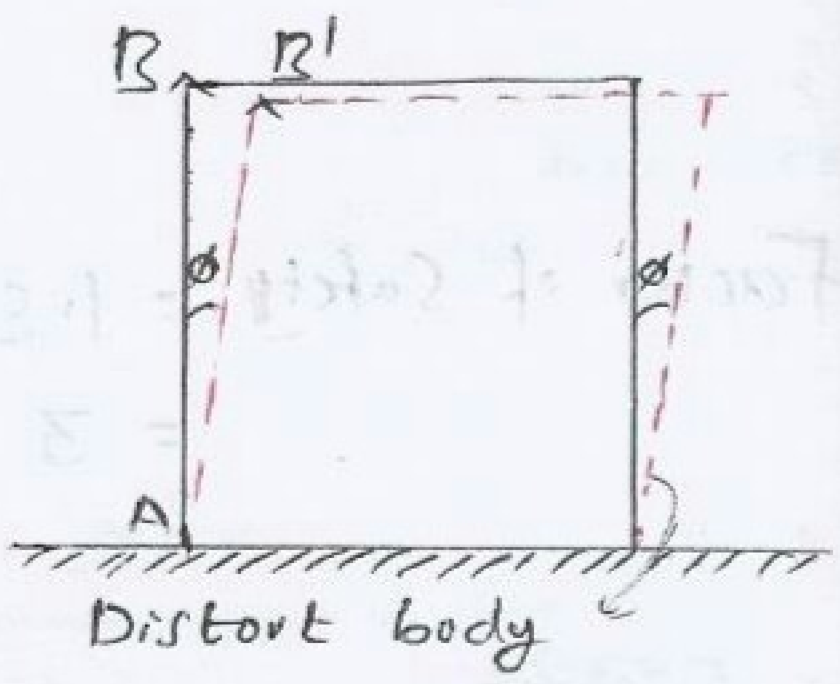
$\Delta D = D \sim D_f$

b) Lateral Strain (e_y)

Strain perpendicular direction of force

$$e_y = \frac{\text{Change in depth}}{\text{Original depth}} = \frac{\Delta D}{D}$$

2. Shear Strain (ϕ)



$$\phi = \frac{BB'}{AB}$$

Is the angle through which the body distors (twist out of shape) under the application of shear load.

3. Volumetric Strain (e_v)

$$e_v = \frac{\Delta V}{V} = \frac{\text{Change in volume}}{\text{initial volume}}$$

$$\boxed{e_v = \frac{3\sigma}{E}(1-2\mu) = \frac{\sigma}{k}}$$

$$\boxed{e_v = e_x + e_y + e_z}$$

4. Superficial Strain (or) Area Strain (e_A)

$$e_A = \frac{\Delta A}{A} = \frac{\text{Change in area}}{\text{initial area}}$$

$$\boxed{e_s = e_A = e_x + e_y}$$

$\because A = LD \Rightarrow dA = D(dL) + L(dD)$

$$\frac{dA}{A} = \frac{D(dL)}{A} + \frac{L(dD)}{A}$$

$$\frac{\Delta A}{A} = \frac{\Delta L}{L} + \frac{\Delta D}{D} = e_x + e_y$$

$\therefore$ Superficial strain = lateral strain(e_y) + longitudinal strain(e_x)

* Stretch (or) Extension ratio (λ)

$$\lambda = \frac{\text{Final dimension}}{\text{Initial dimension}}$$

$\lambda = \frac{L_f}{L}$ = Elongation ratio $= 1-e$

$\lambda = \frac{D_f}{D}$ = Stretch ratio $= 1+e$

Elastic Constants

Hooke's law:

a) Stress ∝ Strain ⇒ $\sigma = E \cdot e$

b) Shear stress ∝ Shear strain ⇒ $\tau = G\phi$

1. Young's Modulus (E)

Used to ~~to~~ Characterize material, also called as "Modulus of Elasticity"

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{e}$$

2. Shear Modulus (G)

Used to measure the stiffness of the material. "Modulus of Rigidity".

$$G = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

3. Bulk modulus (k)

When a body subjected to "Like & Equal" Normal stress (σ), along three mutually ⊥r directions.

$$k = \frac{\text{Normal stress}}{\text{Volumetric strain}} = \frac{\sigma_n}{e_v}$$

* Poisson's Ratio (μ)

$$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{e_y}{e_x}$$

(μ lie b/n 0 & 0.5)

* Relationship between E, G, k and μ

$$E = 2G(1+\mu) \qquad E = 3k(1-2\mu)$$

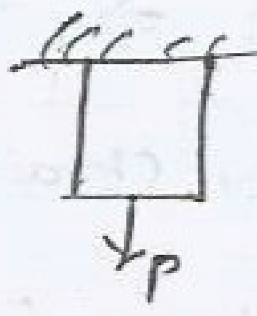
$$E = \frac{9kG}{3k+G} \qquad \mu = \frac{3k-2G}{6k+2G}$$

Type of material	No. of Independent Elastic Constants
Isotropic	2 (E & μ)
Orthotropic	9
Anisotropic	21

Elongation of bodies

1. Elongation of body due to External load (Self wt Ignored)

$$\Delta L = \frac{PL}{AE}$$



$$\left(\because E = \frac{\sigma}{e} = \frac{P/A}{\Delta L/L} \right)$$

2. Due to Self wt only

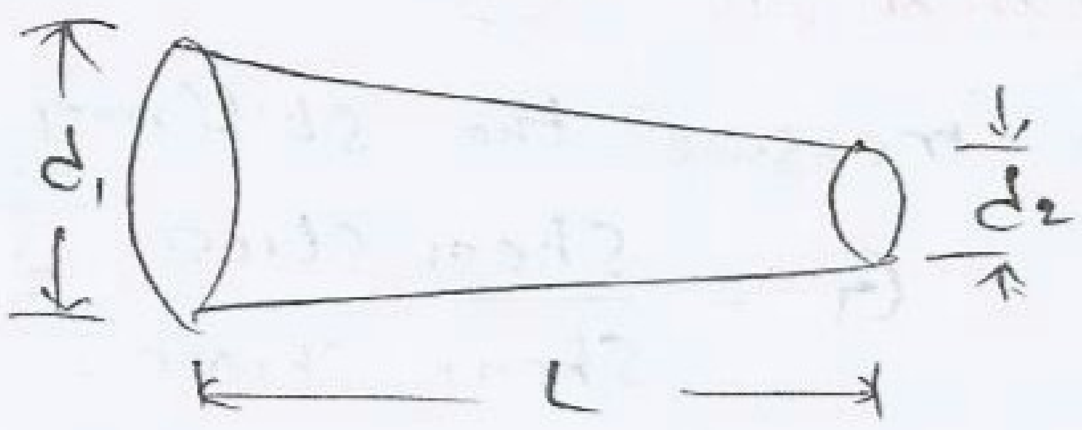
$$\Delta L = \frac{WL}{2AE}$$

$$\Delta L = \frac{\gamma L^2}{2E} \quad (\text{For prismatic bar})(W = AL\gamma)$$

$$= \frac{\gamma L^2}{6E} \quad (\text{For Conical bar})\left(\because W = \tfrac{1}{3}AL\gamma\right)$$

3. Tapered body

$$\Delta L = \frac{4PL}{\pi E d_1 d_2}$$



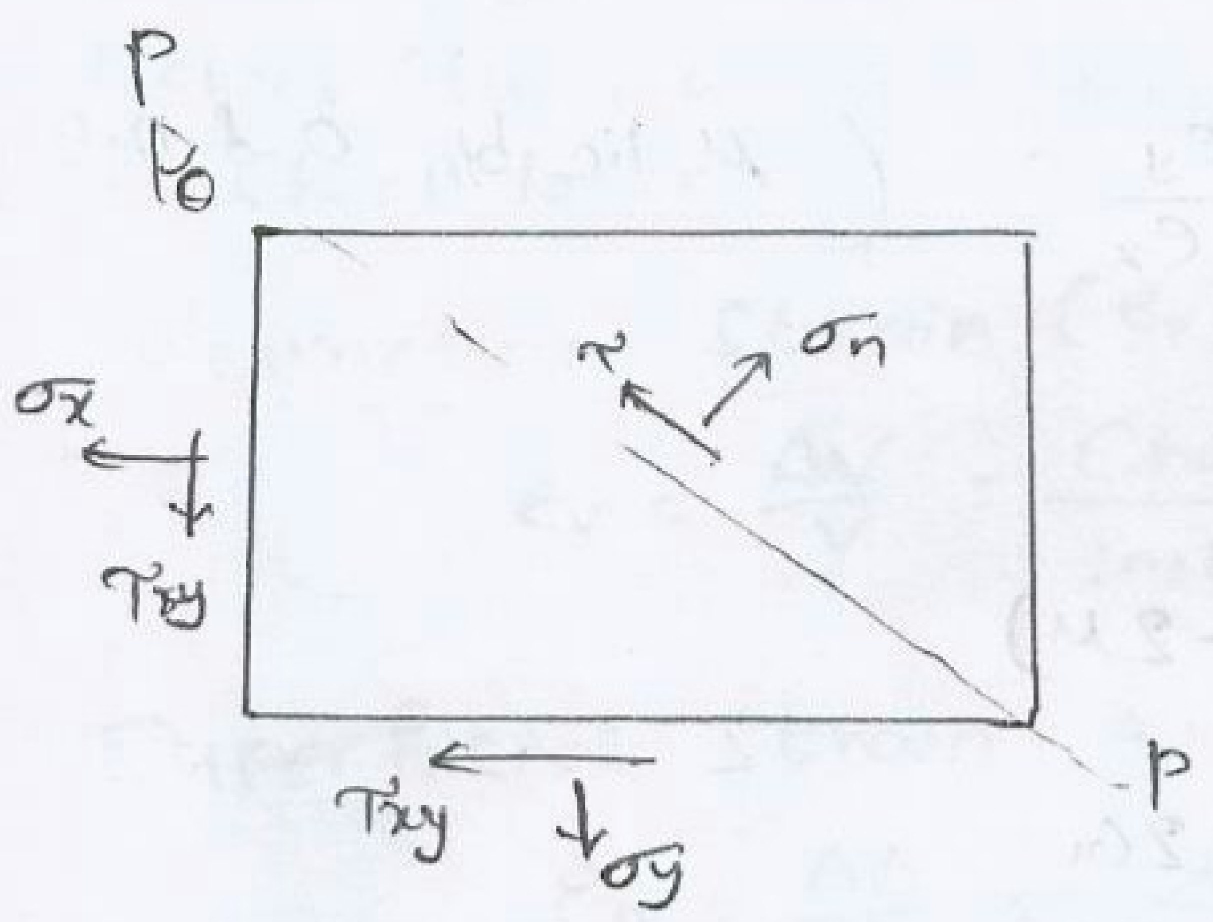
4) Due to Temperature

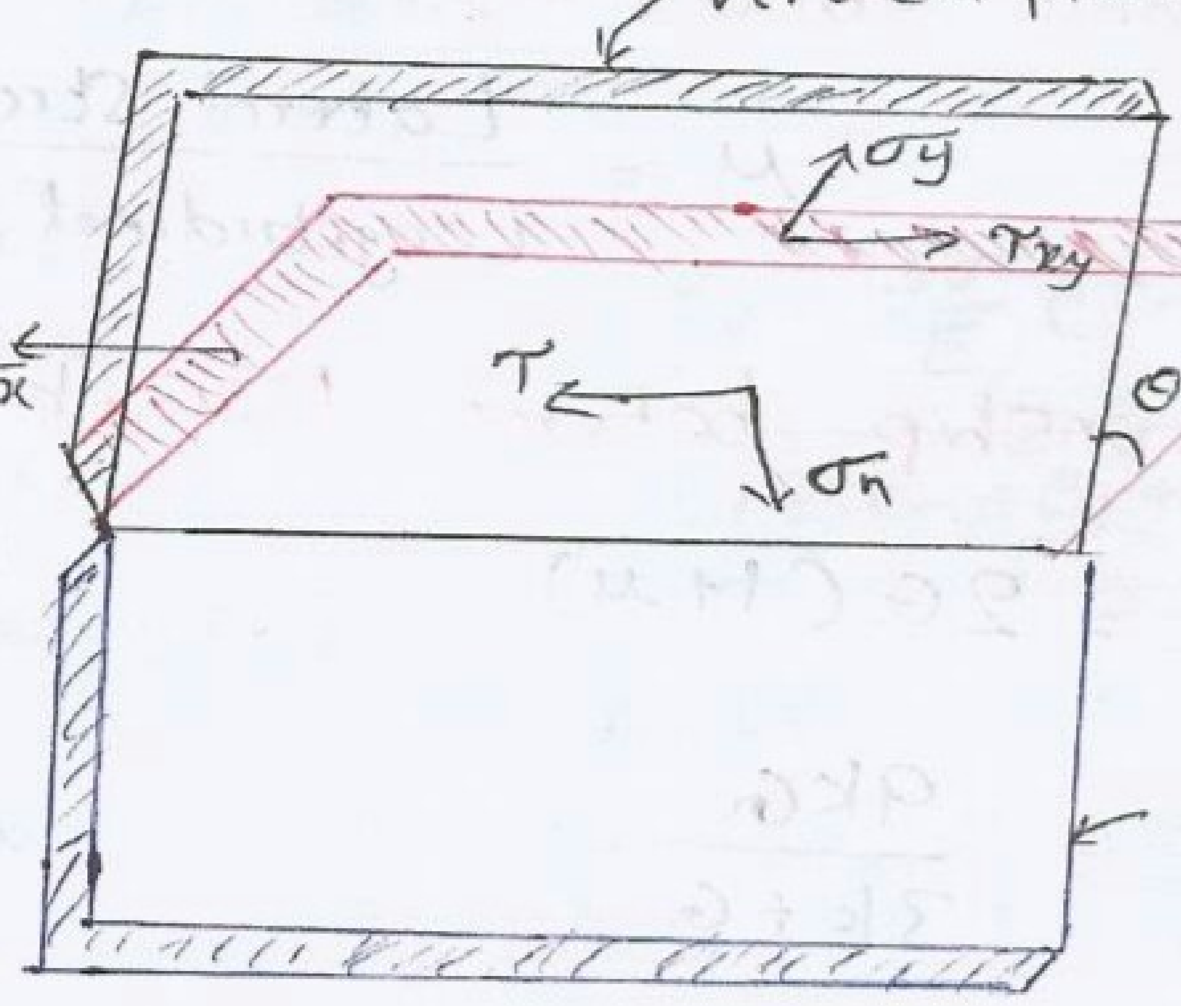
$$\Delta L = L\alpha\Delta T$$

Thermal stress $\sigma_T = E\alpha\Delta T$
Thermal Strain $e_T = \alpha\Delta T$

Compound Stresses on Planes

In planes Consider Stresses & Strains are negligible in Z-direction.





σ_x, σ_y = principle stresses in the direction x & y

T_{xy} = Shear stress is acting on x-face & in y-direction.

Combined Stresses

σ_n = Normal stress (i.e the Combined stress Normal to the surface)

τ = Shear stress (i.e The combined stress tangential to the surface)

$R = \sqrt{\sigma_n^2 + \tau^2}$ (i.e Resultant stress, θ angle to the surface)

Formulas

$$\sigma_n = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + T_{xy}\sin 2\theta$$

$$\tau = \left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + T_{xy}\cos 2\theta.$$

Special cases

1. Uniaxial Stress (only σ_x acts) ($\sigma_y = 0$ & $T_{xy} = 0$)
2. Biaxial stress (σ_x & σ_y acts) ($T_{xy} = 0$)
3. pure Shear (only T_{xy} acts) ($\sigma_x = \sigma_y = 0$)

Principal Stresses

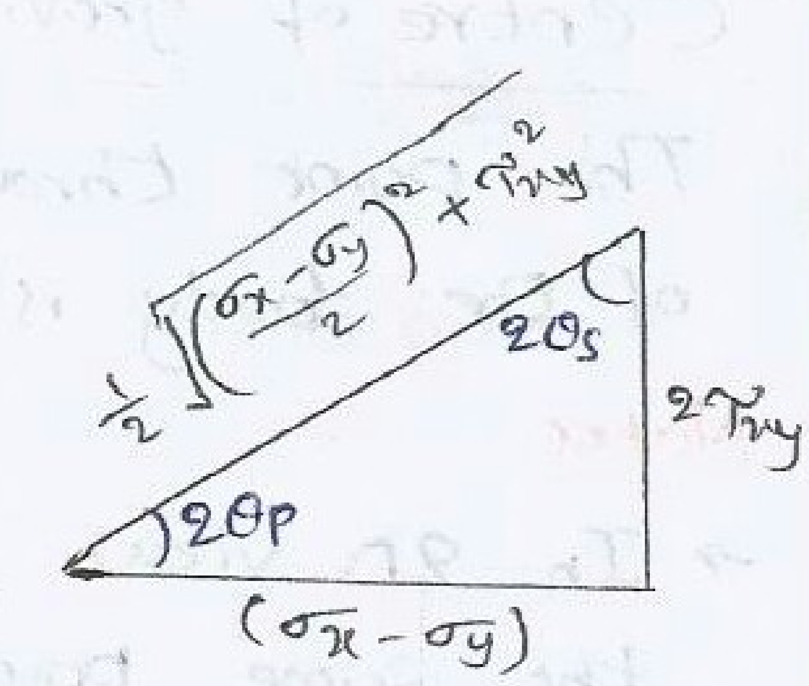
Since σ_n, will be max at $\frac{d\sigma_n}{d\theta}=0$, i.e at $\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x-\sigma_y)}$

(θ_p = Angle b/n max Normal stress with vertical plane. It is the function of 'Tan').

Hence σ_n is max at $2\theta_p$ (or) $\pi+2\theta_p \Rightarrow \theta_{p_1}-\theta_{p_2}=\pi/2$

θ_{p_1} = Major principle stress acting θ_p with verticle

θ_{p_2} = Minor principal stress acting $\frac{\pi}{2}+\theta_p$ with verticle.



Major principal Stress

$$\sigma_{P_1} = \left(\frac{\sigma_x+\sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

Minor principal stress

$$\sigma_{P_2} = \left(\frac{\sigma_x+\sigma_y}{2}\right) - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

∴ The Two principal planes (Major & Minor) differ by angle 90°

Maximum Shear Stress

Since τ, will max at $\frac{d\tau}{d\theta}=0$, i.e at $\tan 2\theta_s = \frac{\sigma_x-\sigma_y}{2\tau_{xy}}$

$||^{ly}$ τ_{max} occur on ⊥r planes (i.e θ_s & $\theta_s+90°$)

$$\therefore \quad \tau_{max} = \frac{\sigma_{P_1}-\sigma_{P_2}}{2}$$

$$= \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$\because \tan 2\theta_p \times \tan 2\theta_s = 1$

$\tan 2\theta_p = \cot 2\theta_s$

$\tan 2\theta_p = \tan(90 \pm 2\theta_p)$

$2\theta_p - 2\theta_s = \pm 90°$

$\theta_p = \theta_s \pm 45°$

∴ Planes of principle stresses (σ_p) & Max shear stresses (τ_{max}) are inclined 45° to each other.

Mohr's Circle

It is the graphical representation of Compound stresses. (Formulas are same)

$\sigma_p \rightarrow$ +ve

τ ↻ +ve

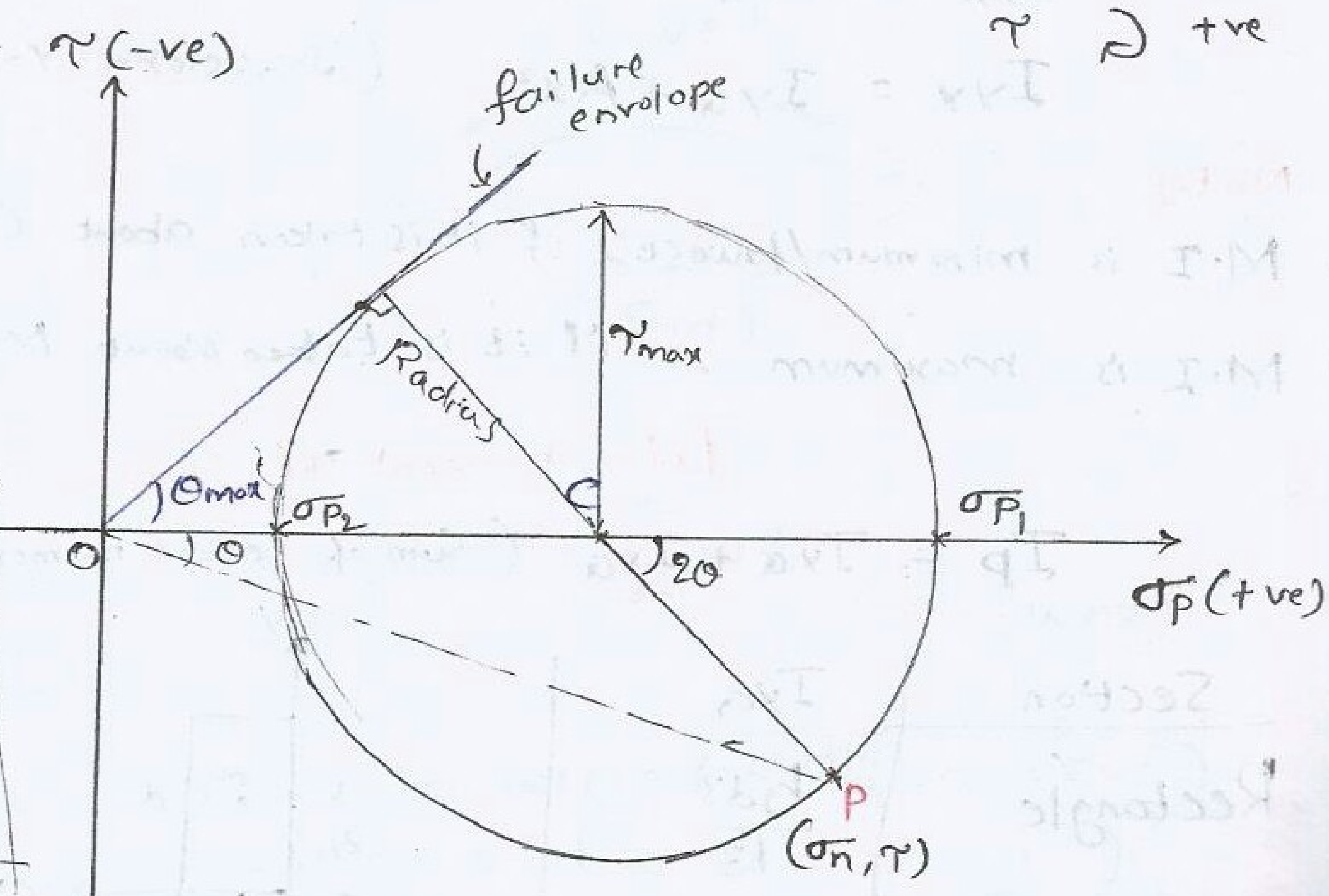
Centre $= \left(\frac{\sigma_{p_1}+\sigma_{p_2}}{2}, 0\right)$

$= \left(\frac{\sigma_x+\sigma_y}{2}, 0\right)$

Radius = τ_{max}

$\cancel{\theta_{max} = \frac{\left(\frac{\sigma_{P_1}-\sigma_{P_2}}{2}\right)}{\text{Radius}}}$

$\cancel{= \frac{\sigma_x-\sigma_y}{2\tau_{max}}}$



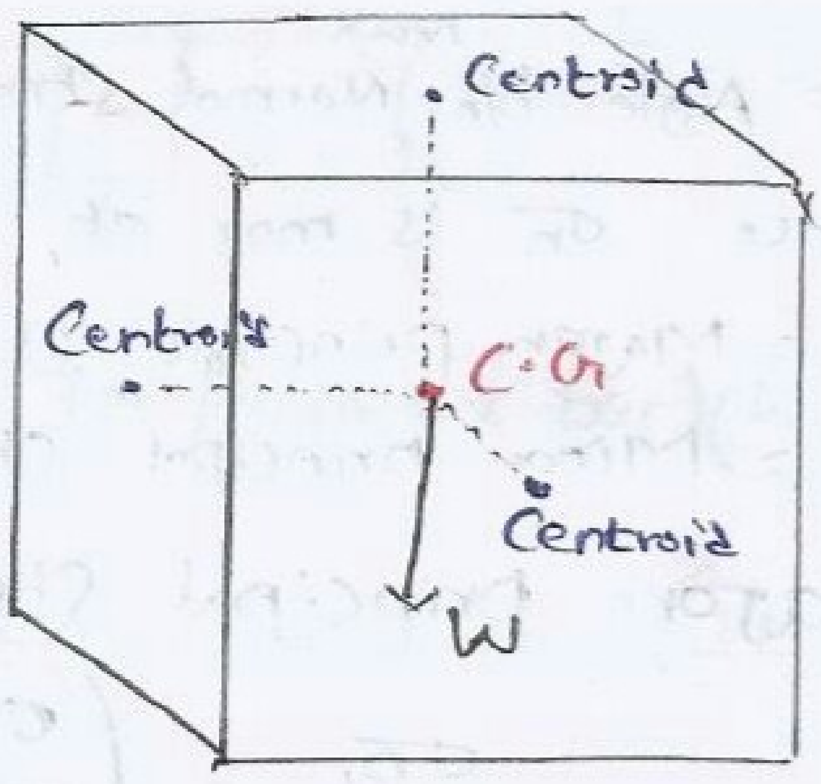
$$\theta_{max} = \frac{\text{Radius}}{\left(\frac{\sigma_{P_1}+\sigma_{P_2}}{2}\right)} = \frac{\tau_{max}}{\left(\frac{\sigma_x+\sigma_y}{2}\right)}$$

($\because \sigma_y$ is unlike, i.e eg. $\sigma_y = -20 \Rightarrow \sigma_x - \sigma_y = \sigma_x - (-20) = \sigma_x + 20 = \cancel{\sigma_x+\sigma_y}$)

Centroid & Centre of Gravity (C.G)

Centroid:

The point where whole are of the figure is assumed to be Concentrated.



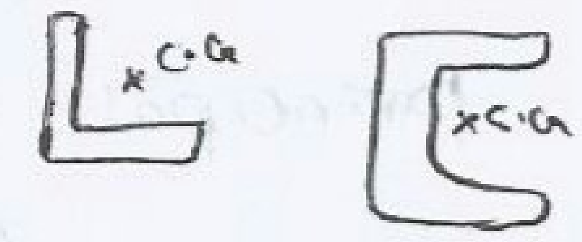
Centre of gravity (C.G)

The point through which the whole weight of the body is assumed to act.

Notes:

→ In 2D view the Centroid & C.G are located at the same point.

→ The Centroid may be lie outside the section (eg.

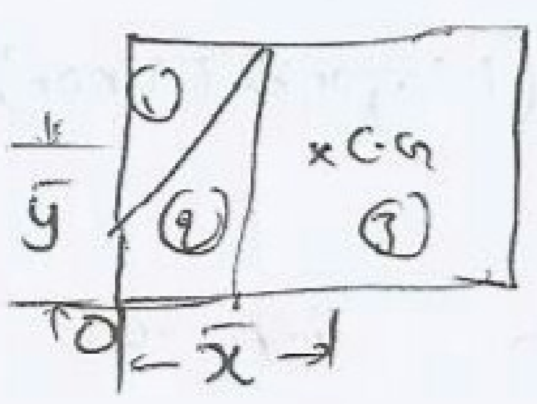


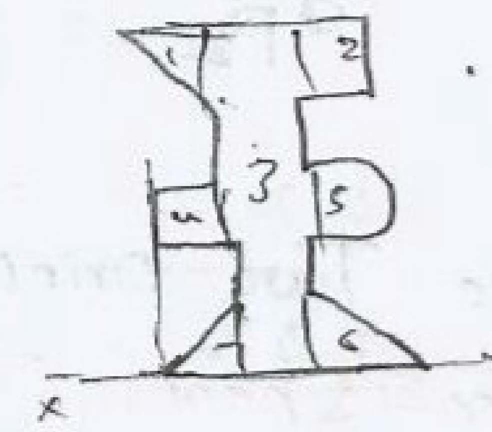
)

Centroid of lamina consisting of different areas

$$\bar{x} = \frac{\Sigma A\bar{x}}{\Sigma A}$$

$$\bar{y} = \frac{\Sigma A y}{\Sigma A}$$





→ The C.G of rectangle lies at a point where its two diagonal meet

→ A body may be regular / Irregular it has only one C.G.

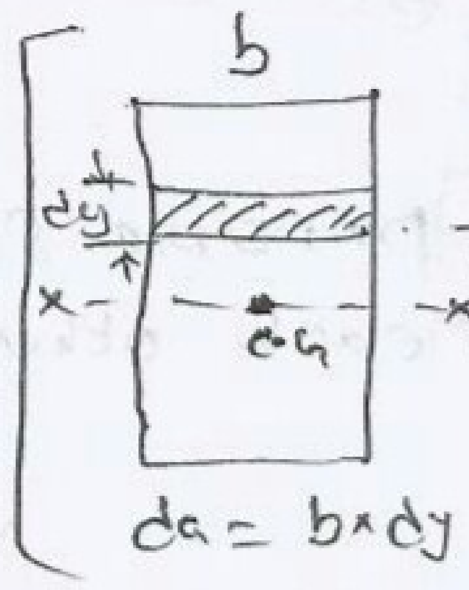
Moment of Inertia

Second moment of area

also known as "Moment of inertia of plane area".

Is for single known symmetrical geometric body

$$I_{xG} = \int y^2 da \quad \& \quad I_{yG} = \int x^2 da$$

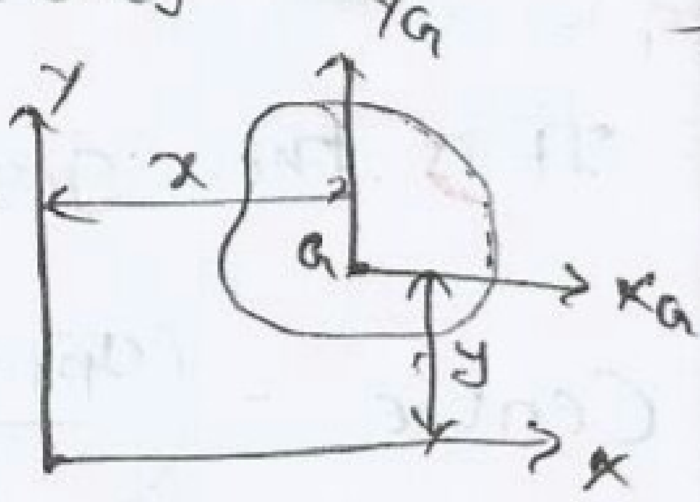


$$\left[\int_{-d/2}^{d/2} y^2 \times b\, dy \right] \quad I_x = \frac{bd^3}{12}$$

Parallel Axes Theorem

$$I_{xx} = I_{xG} + Ay^2 \quad (I_{xG} \text{ relative x-axis})$$

$$I_{yy} = I_{yG} + Ax^2 \quad (I_{yG} \text{ relative Y-axis})$$



Notes:

→ M.I is minimum/lowest, if it is taken about C.G (i.e. $x=0$ & $y=0$ ⇒ $I_{xx} = I_{xG}$, $I_{yy} = I_{yG}$)

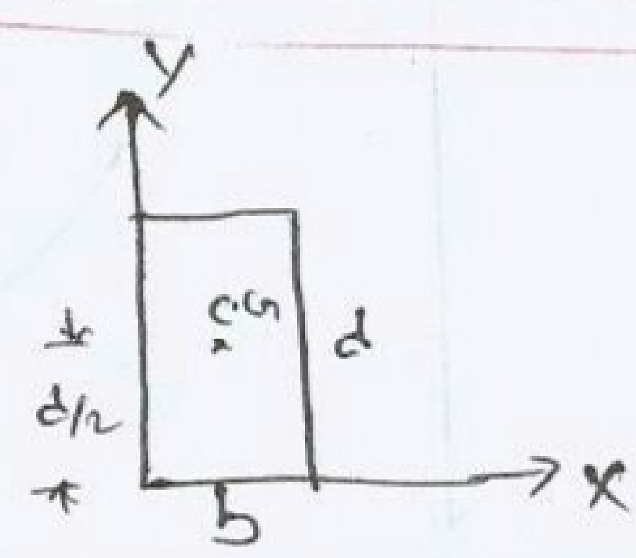
→ M.I is maximum, if it is taken about Major principal axis ($= I_{xx}$ if $y > x$, $= I_{yy}$ if $x > y$)

Polar moment of Inertia

$$I_p = I_{xG} + I_{yG} \quad (\text{Sum of Second moment of area about X \& Y axis})$$

Section	I_{xG}
Rectangle	$\frac{bd^3}{12}$
Triangle	$\frac{bh^3}{36}$
Circle	$\frac{\pi d^4}{64}$

(origin of axis at C.G)

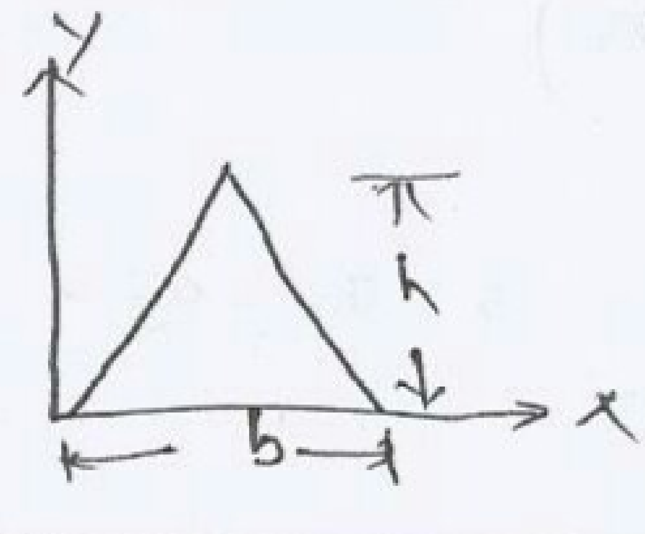


(origin of axis at corner)

$$I_{xx} = \frac{bd^3}{3}, \quad I_{yy} = \frac{db^3}{3}$$

$$\left(\frac{bd^3}{12} + (bd) \times \left(\frac{d}{2}\right)^2 = \frac{bd^3}{3} \right)$$

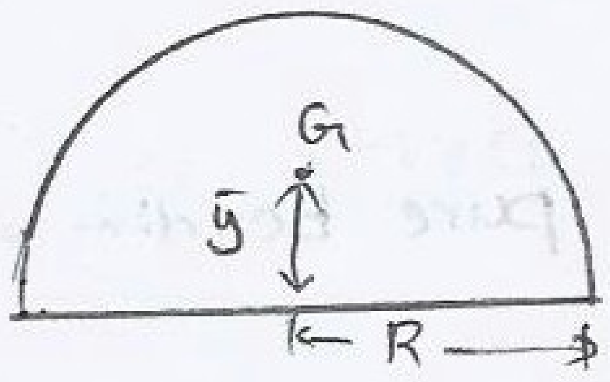
(origin of axis at vertex)



$$I_{xx} = \frac{bh^3}{12}$$

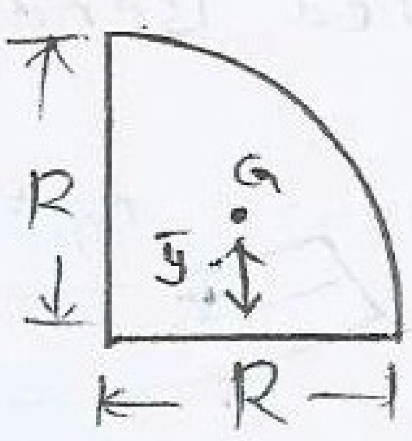
$$\left(\frac{bh^3}{36} + \left(\frac{1}{2}bh\right)\left(\frac{1}{3}h\right)^2 = \frac{bh^3}{12} \right)$$

Semicircle

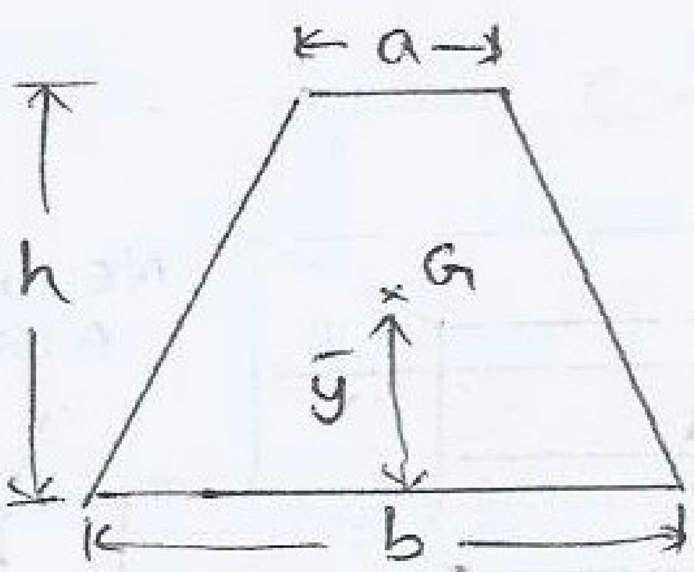


$\bar{y} = \frac{4R}{3\pi}$

Quadrant Circle.



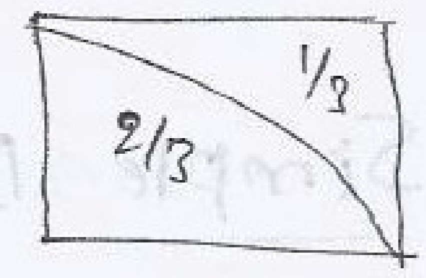
$\bar{y} = \frac{4R}{3\pi}$

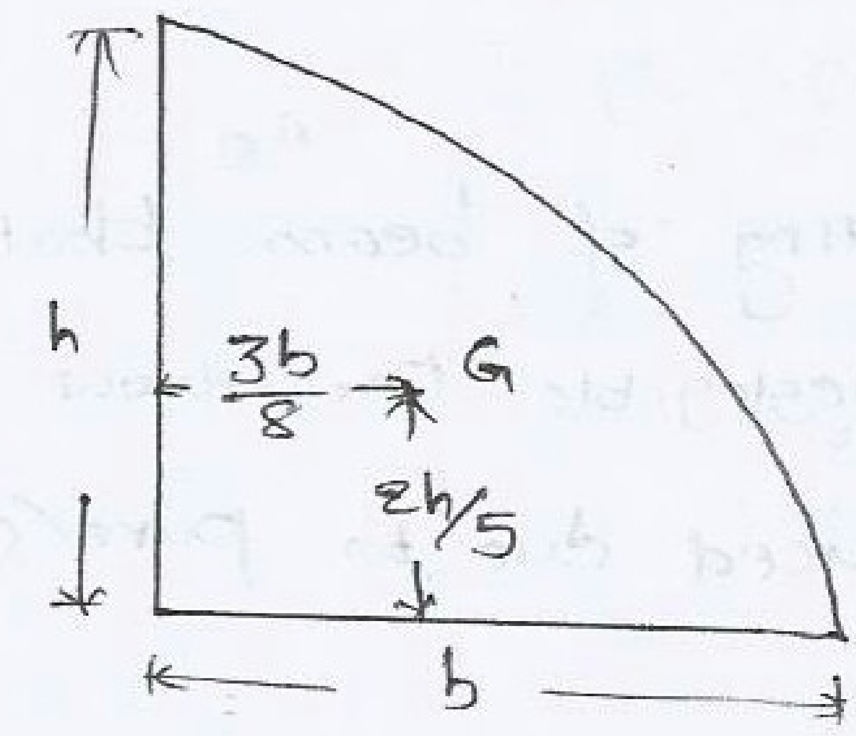


$\bar{y} = \frac{h}{3}\left(\frac{b+2a}{b+a}\right)$

$A = \frac{1}{2}h(a+b)$

Parabolic Segment



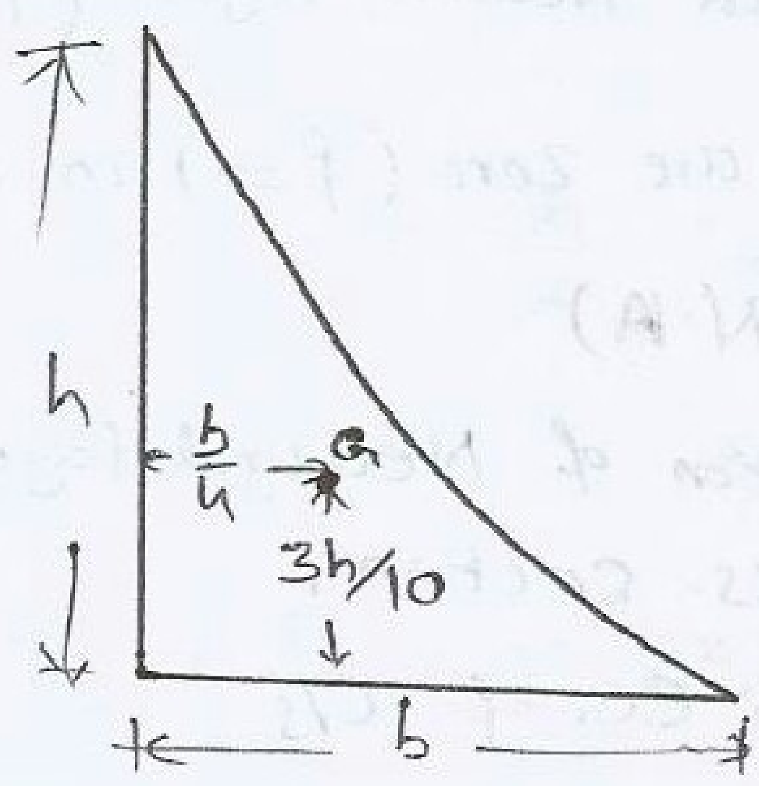


$A = \frac{2}{3}bh$

$\left(\frac{3}{8}, \frac{2}{5}\right)$

least side

Parabolic Spondrel



$A = \frac{1}{3}bh$

$\left(\frac{1}{4}, \frac{3}{10}\right)$

least side

Shear Force & Bending Moment Diagrams

Shear Force (F)

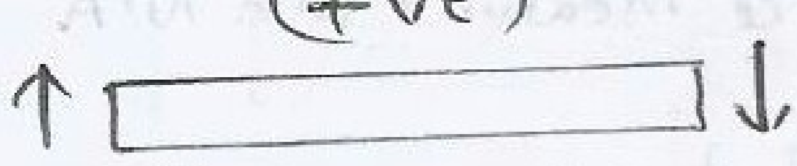
Algebric Sum of all forces on either side of the section.

Bending Moment (M)

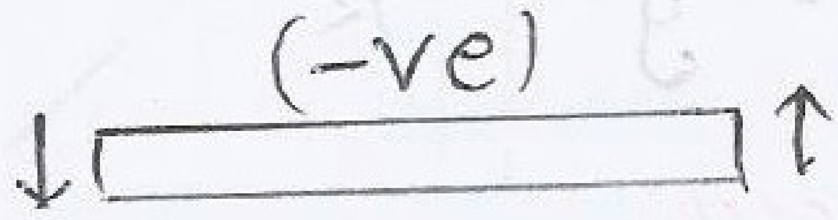
Algebric sum of all the moments of the forces on either side of the section.

Sign Convertions

a) For Shear Force

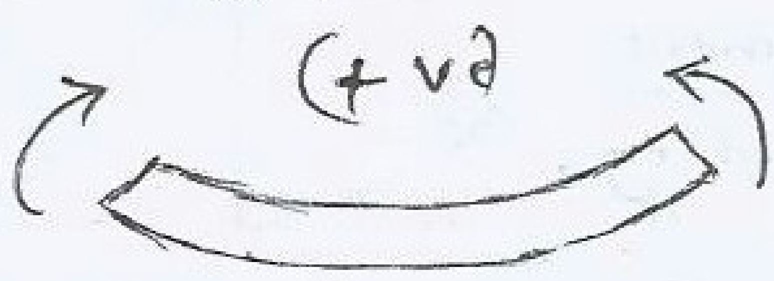


Clockwise Rotation

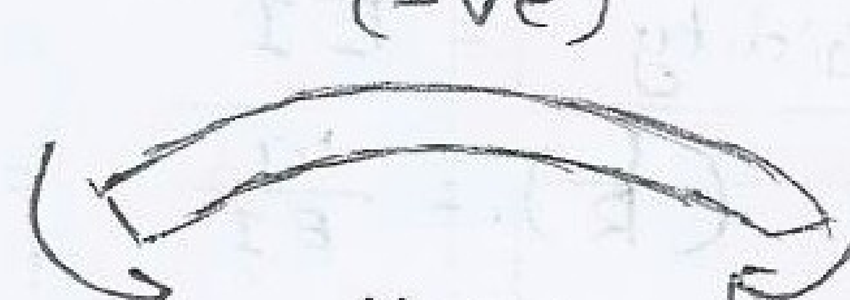


Anti clock wise

b) For Bending Moment



Sagging
(Smile)



Hogging
(Sad)

Relationship between (W, F, M)

$W = \frac{dF}{dx} = \frac{d^2M}{dx^2}$ (or) $\left(F = \frac{dM}{dx}\right)$ $M = \int F\,dx = \iint w\,dx$ $\left(F = \int w\,dx\right)$

Special Conditions

Pure bending : B.M is Constant throughout the section (i.e $\frac{dM}{dx} = 0 \Rightarrow F = 0$)
(It happened for Cantilever couple. → B.M)

Position of Contraflexure . = At Point, where B.M = 0 (or) B.M Changes sign.

B.M is Max when S.F is Zero (or) S.F changes sign

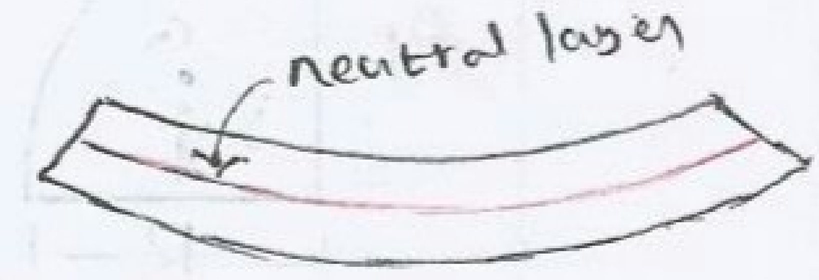
Theory of Simple Bending

Simple Bending

Considering Bending of beam that is accompanied by pure bending only.
(i.e S.F = 0 or Neglegible throughout the section)

∴ The stress induced due to pure/simple bending is called Bending stress.

1. Neutral layer



The layer which is Neither compressed ~~or~~ nor elongated due to Bending is called Neutral layer. (i.e unchanged its length due to Bending)

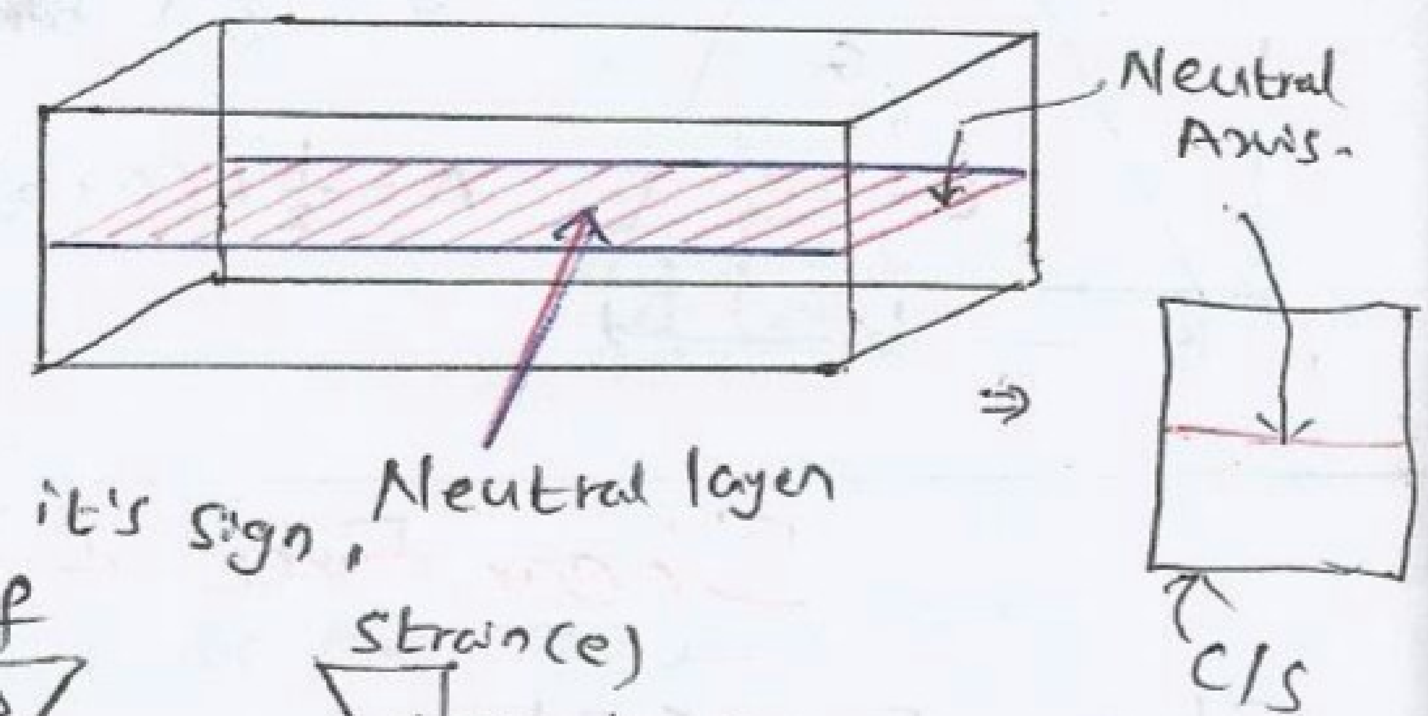
→ Bending stresses are zero ($f=0$) in Neutral layer.

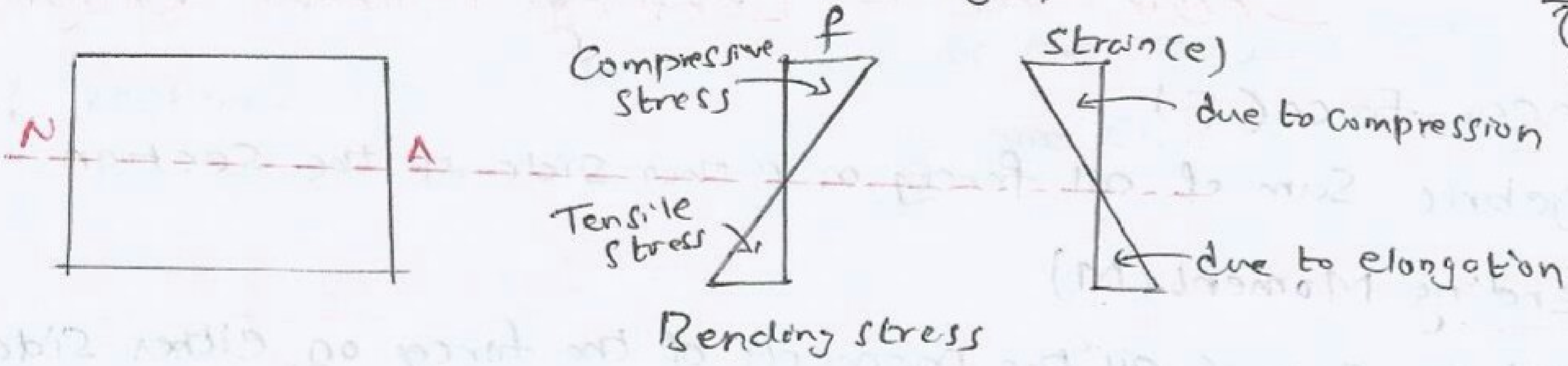
Neutral Axis (N.A)

The line of intersection of Neutral layer with plane of cross-section.

→ It passes through CG of C/S

→ The axis where the stress & strain changes it's sign.

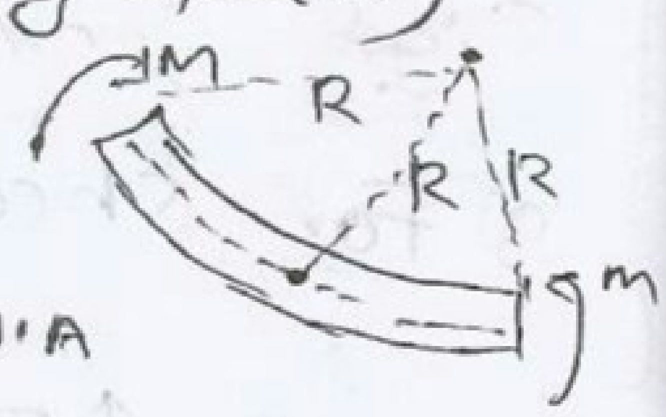




Equation of pure Bending (Also called Bernoulli-Euler bending equation)

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

R = Radius of curvature (from N.A to centre)
y = Distance measured from N.A



Important terms

Section modulus $(Z) = \frac{I}{y}$ $\Rightarrow \left(f = \frac{M}{Z}\right)$

Flexural Rigidity $= EI$

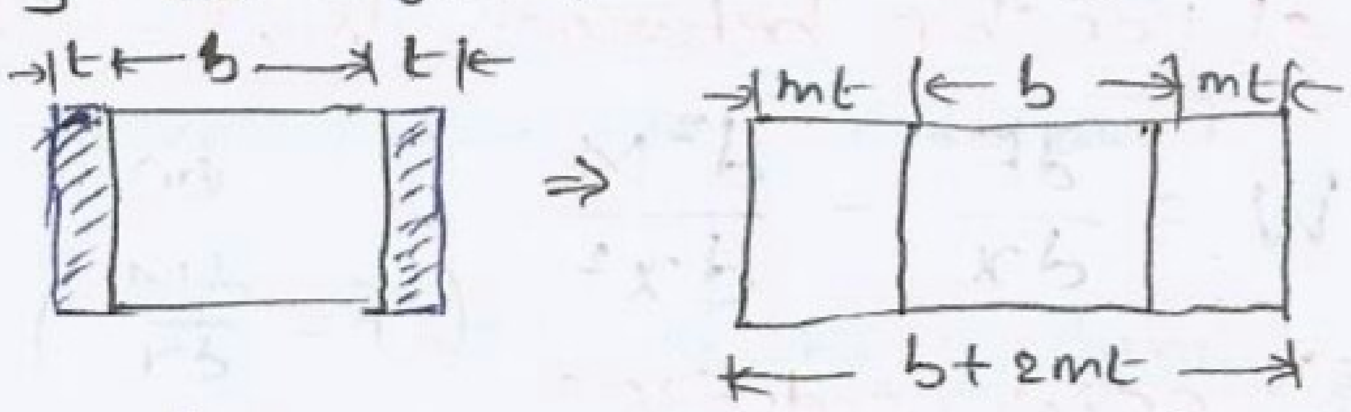
Curvature $\left(\frac{1}{R}\right) = \frac{M}{EI} = \frac{\text{Bending moment}}{\text{Flexural rigidity}}$.

Beam of uniform ~~Section~~ Strength

The bending stress is constant throughout the beam (i.e $f = \frac{M}{Z}$ = constant)

i.e The Bending stress (f) is also known as Strength of the beam (f).

Beam made of different materials



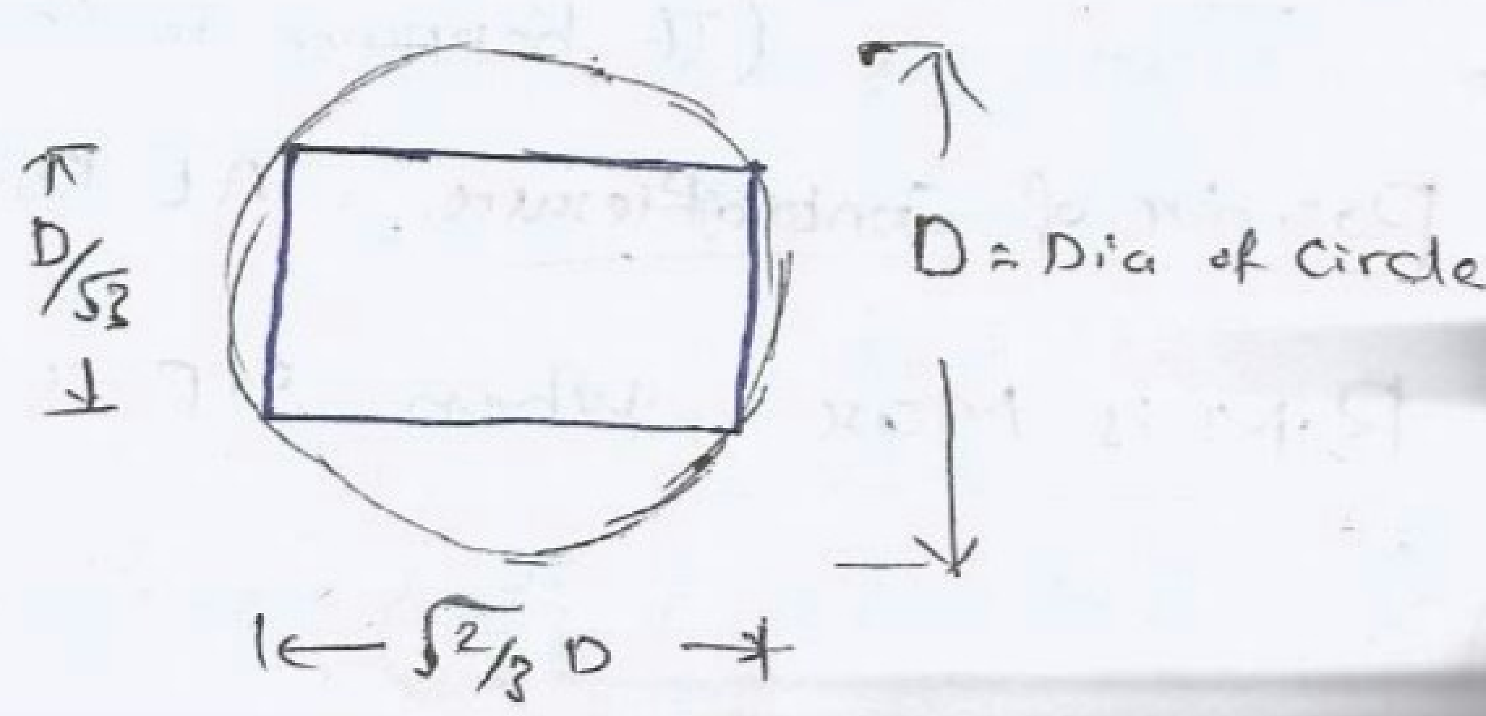
In this case "The width of composite section" is considered as "Equivalent width of section".

∴ Equivalent width of section $= b + 2mt$ (∵ m = modular ratio $= \frac{E_b}{E_t}$)

Max Rectangle cut from Circle

Max Dimensions of Rectangle

$$= \left(\frac{D}{\sqrt{2}}, \sqrt{\frac{2}{3}}\, D\right)$$



Max Dimension of Square $= \frac{D}{\sqrt{2}}$

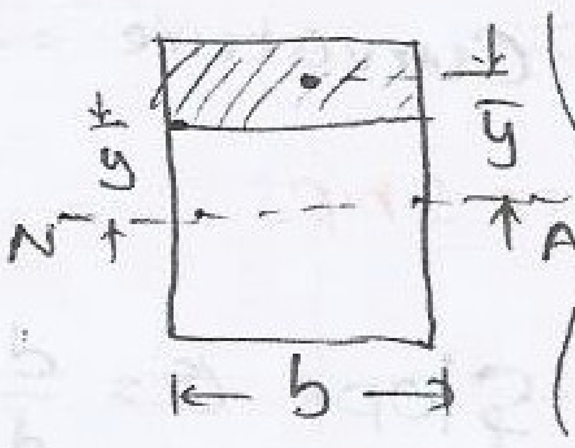
Shear Stress Distribution in Beams

Shear stress at any section $(q) = \dfrac{F(a\bar{y})}{Ib}$

Average shear stress at any $(q_{avg}) = \dfrac{F}{A}$

$\because$ F = S.F at that section
b = width at that section
A = Total area of the C/S
$a\bar{y}$ = Moment of area of the section under consideration about N.A

eg:



a = Area of shaded portion
$\bar{y}$ = Distance from N.A to C.G of shaded

Special Conditions

a) Rectangular section $\left(q_{max} = \frac{3}{2} q_{avg} \right)$

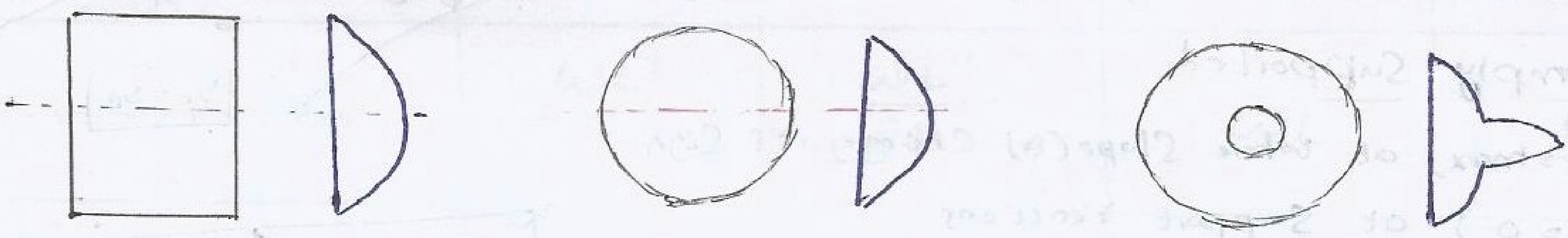
b) Circular section $\left(q_{max} = \frac{4}{3} q_{avg} \right)$

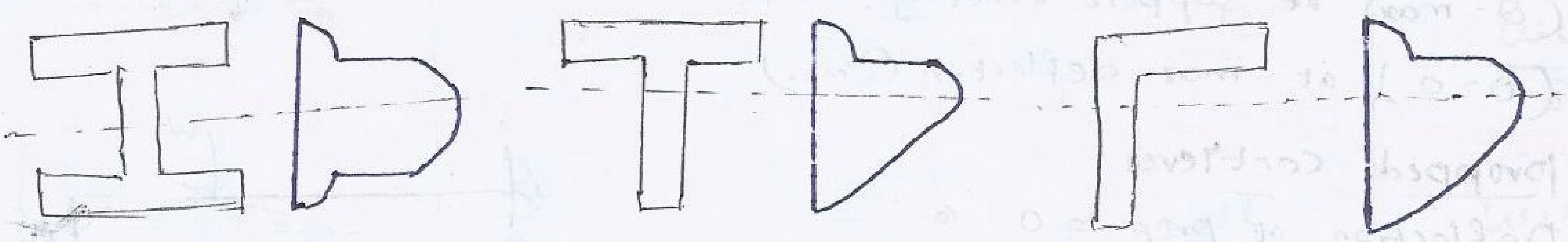
c) Triangular section $\begin{cases} q_{max} = \frac{3}{2} q_{avg} \text{ @ } \frac{h}{2} \text{ from top/bottom} \\ q_{NA} = \frac{4}{3} q_{avg} \end{cases}$

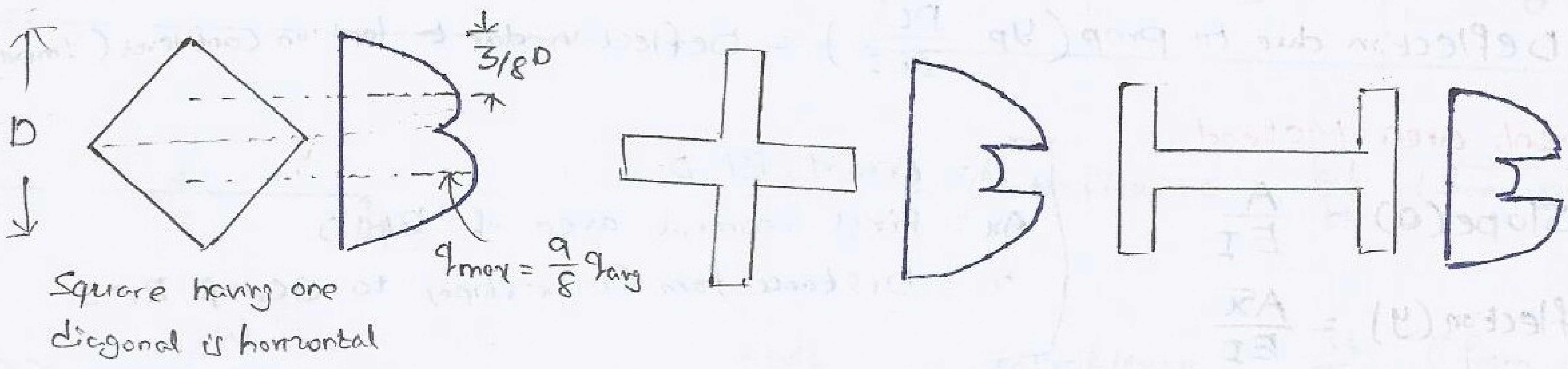
d) I-Section $\left(q_{max} = \frac{F}{8I} \left[\frac{B}{t_w} (D^2 - d^2) + d^2 \right] \right.$

Shear stress distribution for the following sections

a) Rectangle b) Circle c) Hollow circle







Square having one diagonal is horizontal

Important points

→ For pure bending ($F = 0 \Rightarrow$ shear stress also $= 0$)

→ If width (b) is suddenly decrease ⇒ Shear stress (q) suddenly increases by ($\frac{B}{b}$ times) & vise versa.

eg: For I-section, the shear stress at the junction of web & flange suddenly increases from $\frac{F}{8I}(D^2 - d^2)$ to $\frac{B}{t_w}\left[\frac{F}{8I}(D^2 - d^2)\right]$

→ For higher width, shear stress will be low ($\because q \propto \frac{1}{db}$) & vise versa.

→ At extreame edges shear stress must be zero ($\because q \propto a \Rightarrow a = b \times \text{zero} = 0$)

~~Theory of Pure Bending~~

Deflection of Beams (y)

The shape of the deflection of the beam is called — Elastic Line (or) Deflection curve.

The Equation for the Elastic Line is called

$$\text{Curvature} = \frac{1}{R} = \frac{M}{EI} = \frac{d^2y}{dx^2} = \frac{d\theta}{dx}$$

Relationships

$$\text{Slope } \theta = \frac{dy}{dx} \Rightarrow EI\frac{dy}{dx} = \int M\,dx$$

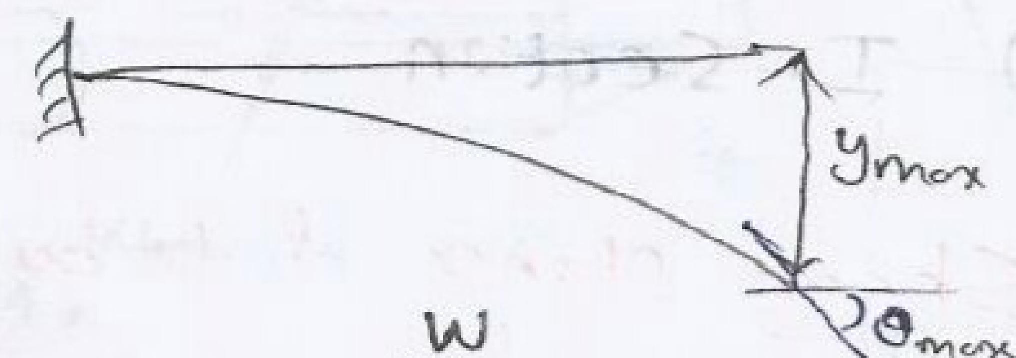
$$\text{Deflection}(y) \Rightarrow EI(y) = \iint M\,dx$$

$$\text{Stiffness }(k) = \frac{\text{Span}\ \cancel{\text{Load}}}{\text{Deflection}} = \frac{\cancel{W}L}{y}$$ (Also depend on (EI))

Important points

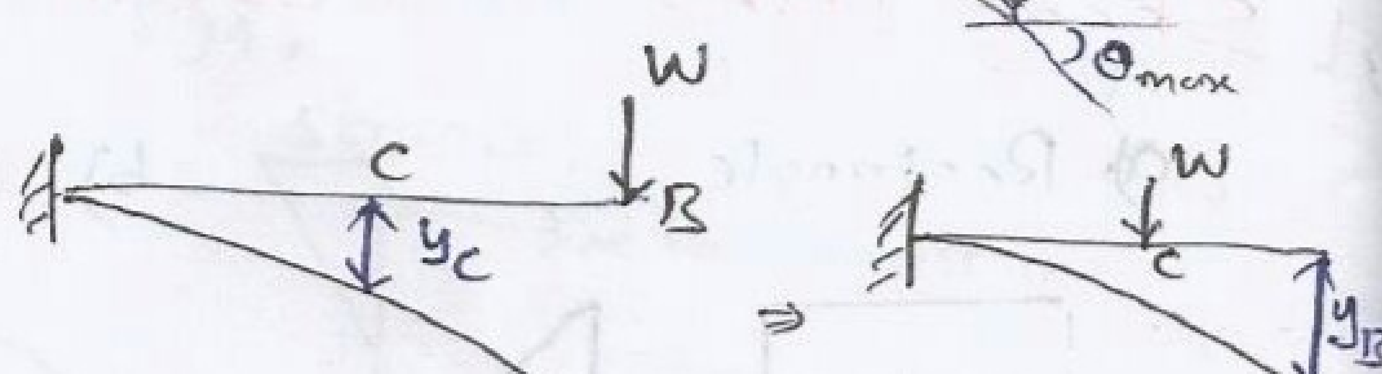
a) Cantilever

y & θ will Maximum at free end, while will zero at fixed end.



(Maxwell's law of reciprocal deflections)

y_c (~~Due~~ Due to load at B) = y_B (Due to load at C)



$\boxed{y_c = y_B}$

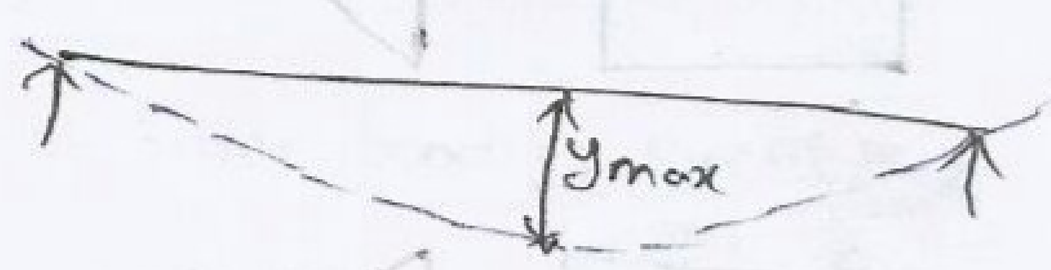
b) Simply Supported

→ (y = max) at where Slope (θ) change it's sign

(y = 0) at Support reactions.

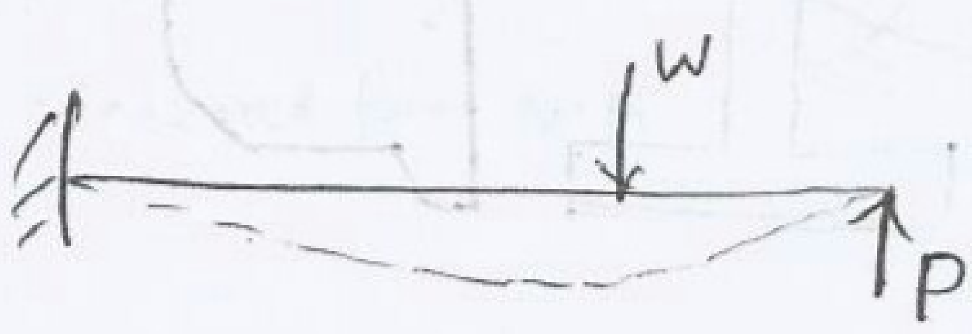
→ (θ = max) at Support reactions, while

(θ = 0) at max deflection (y_{max})



c) propped Cantilever

Deflection at prop = 0 (or)



Deflection due to prop ($y_P = \frac{PL^3}{3EI}$) = Deflection due to load on Cantilever (y_{max})

Moment area Method

$$\text{Slope}(\theta) = \frac{A}{EI}$$

$$\text{Deflection}(y) = \frac{A\bar{x}}{EI}$$

A = Area of BMD

$A\bar{x}$ = First moment area of BMD

$\bar{x}$ = Distance from (max slope) to CG of BMD

Macaulay's Intergration method

eg: $\int (x-a)^2 dx = \frac{(x-a)^3}{3} + c$ (or) $\int (x-a)\,dx = \frac{(x-a)^2}{2} + c$

Double Integration method

eg: $\int (x-a)^2 dx = \int (x^2 - 2ax + a^2)\,dx = \left(\frac{x^3}{3} - \frac{2ax^2}{2} + ax\right) + c$ (or) $\int (x-a)\,dx = \left(\frac{x^2}{2} - ax\right) + c$

[∵ Constants will be find out by applying boundary Conditions]

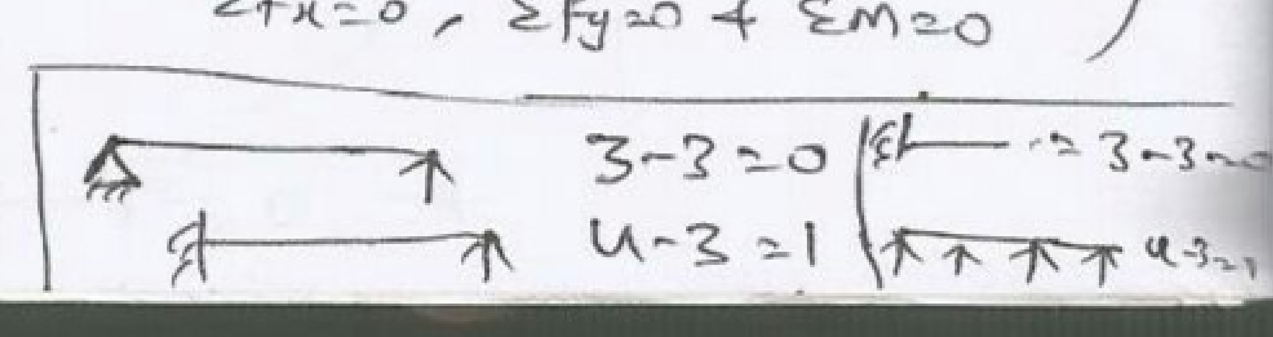
Constants will be add to the 1st term only

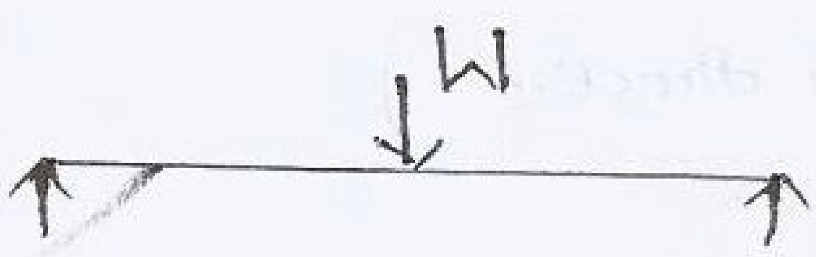
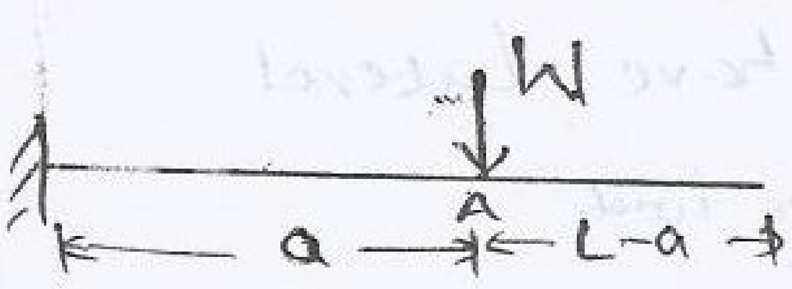
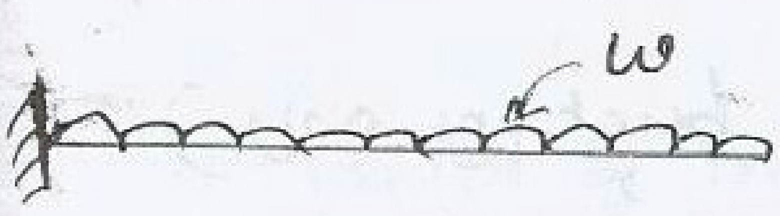
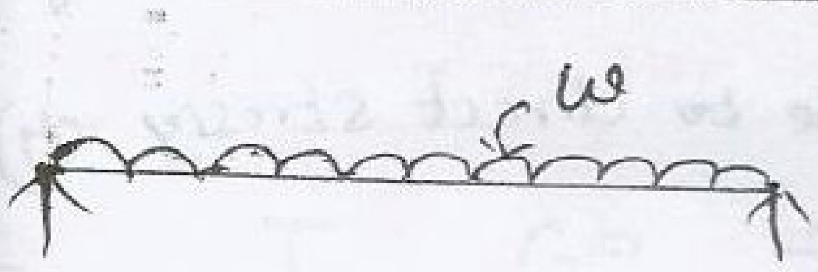
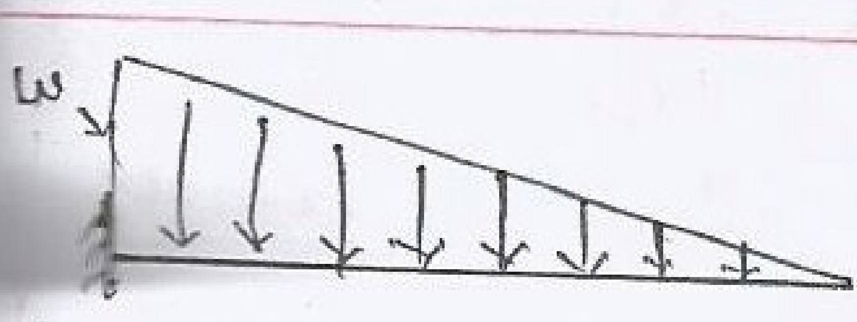
---

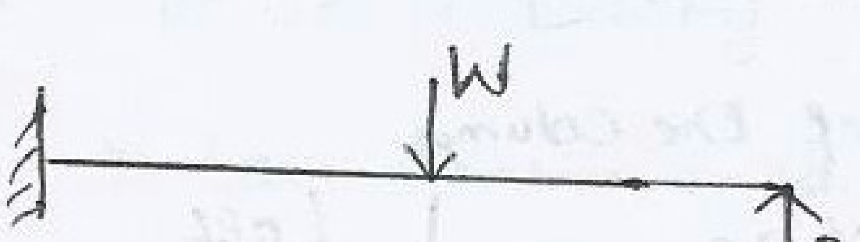
*) Static Indeterminacy = No. of unknown reacts − 3 (Available Equilibrium equations $\Sigma f_x = 0$, $\Sigma f_y = 0$ & $\Sigma M = 0$)

→ No. of unknown reacts ≤ 3 (Determinate beam)

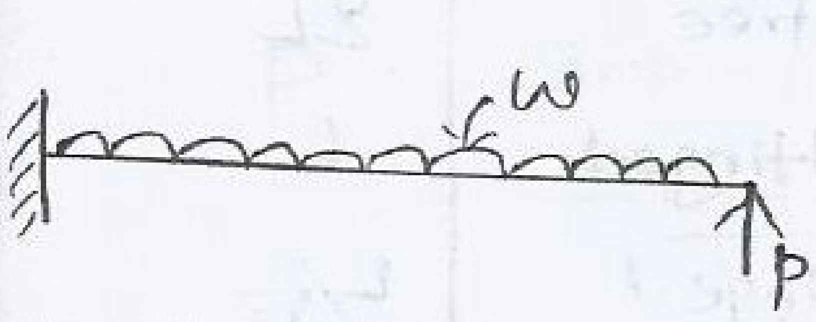
No. of unknown reactions > 3 (Indeterminate beam)



Type of Loading (L = Span length)	Max Slope (θ_{max})	Max. Deflection (y_{max})	Max S.F (F)	Max B.M (M)
	$\frac{ML}{EI}$	$\frac{ML^2}{2EI}$	0	M
	$\frac{ML}{8EI}$	$\frac{ML^2}{24EI}$	$\frac{M}{L}$	M
	$\frac{WL^2}{2EI}$	$\frac{WL^3}{3EI}$	W	WL
	$\frac{WL^2}{16EI}$	$\frac{WL^3}{48EI}$	$\frac{W}{2}$	$\frac{WL}{4}$
	$\theta_A = \frac{Wa^2}{2EI}$	$y_A = \frac{Wa^3}{3EI}$	W	Wa
	$\frac{wL^3}{6EI}$	$\frac{wL^4}{8EI}$	wL	$\frac{wL^2}{2}$
	$\frac{wL^3}{24EI}$	$\frac{5wL^4}{384EI}$	$\frac{wL}{2}$	$\frac{wL^2}{8}$
	$\frac{wL^3}{24EI}$	$\frac{wL^4}{30EI}$	$\frac{wL}{2}$	$\frac{wL^2}{6}$



$P = \frac{5W}{16}$ & Contraflexure $= \frac{8L}{11}$ (from prop)



$P = \frac{3wL}{8}$ & Contraflexure $= \frac{3L}{4}$ (from prop)

Columns & Struts

Strut

Is a Compression member in any position other than vertical.
→ Generally these are used to Connect/support main members.

Column

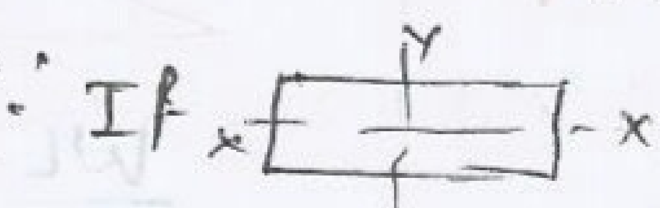
Is a vertical Compression member used in Buildings.

Slenderness Ratio

$$\lambda = \frac{\text{Effective length}}{\text{Least radius of gyration}} = \frac{L_{eff}}{r_{min}}$$

∵ Radius of gyration (r)

$$r = \sqrt{\frac{I}{A}} \Rightarrow \text{least radius of gyration } (r_{min}) = \sqrt{\frac{I_{min}}{A}}$$

(∵ If  $r_{yy} < r_{xx}$ then Column will fail in YY direction)

Buckling (or) Crippling load

The maximum limited load at which the column tends to have Lateral displacement (or) Tends to buckle is known as buckling load.

Classification of Columns

(d = least lateral dimension of column)

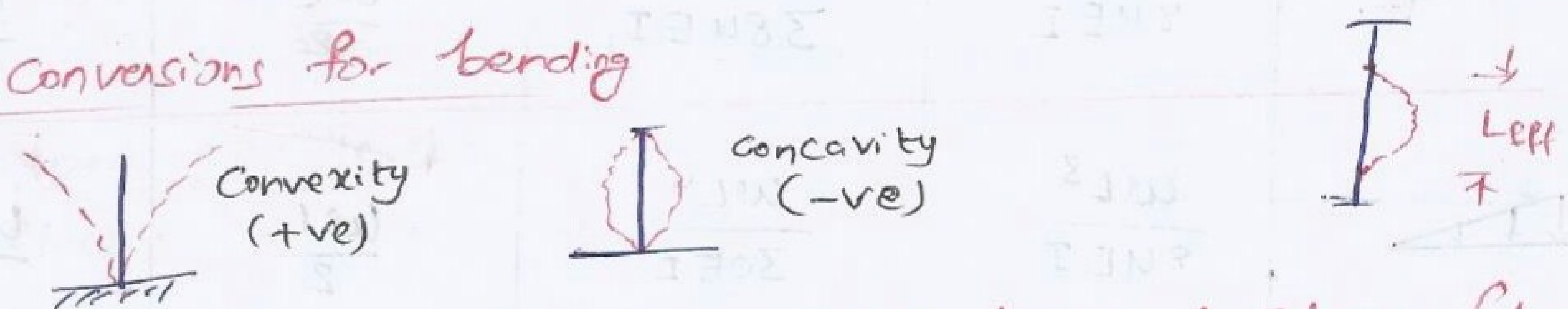
a) Long columns

$L > 30d$ and $\lambda > 120$ (long columns fails due to buckling only)

b) Short Columns

$L < 8d$ and $\lambda < 80$ (Short columns fail due to direct stresses only)

Sign Conventions for bending



Equivalent (or) Effective (or) Unsupported length of Column (L_{eff})

The length at which the Column Can resist the Compressive loads w.r.t end Condns.
→ Is the length b/n points of Contraflexure of Curvature of the Column

End Condition	L_{eff}
1) Upper end Free	$2L$
2) Both ends Hinged	L
3) Upper end hinged lower end fixed	$L/\sqrt{2}$
4) Both ends Fixed	$L/2$

Eulers formula (For long columns only)

Crippling load $\boxed{(P_{cr}) = \frac{\pi^2 EI}{L_{eff}^2}}$

Rankine's Formula (Any type of Column)

$$\frac{1}{P_R} = \frac{1}{P_{cr}} + \frac{1}{P_c}$$

$$P_R = \frac{f_y A}{1+\alpha\lambda^2}$$

∵ P_c = Crushing load = $f_y \times A$
α = Rankine's Coefficient = $\frac{f_y}{\pi^2 E}$

Notes

→ Based on theoritical considerations, Equilateral triangle is the optimum cross-section shape of the Column.

$$\text{Safe load} = \frac{\text{Crippling load } (P_{cr})}{\text{Factor of safety}}$$

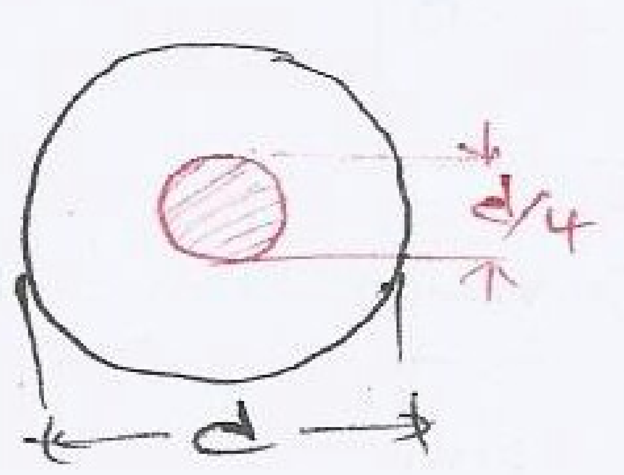
	FOS
1) wrought iron, Mild steel & medium steel	3
2) Cast Iron	5
3) Timber	6

Core of a Cross-Section

The part of cross-sectional area within which a direct load is applied, tensile stress will not be developed any where in the cross-section

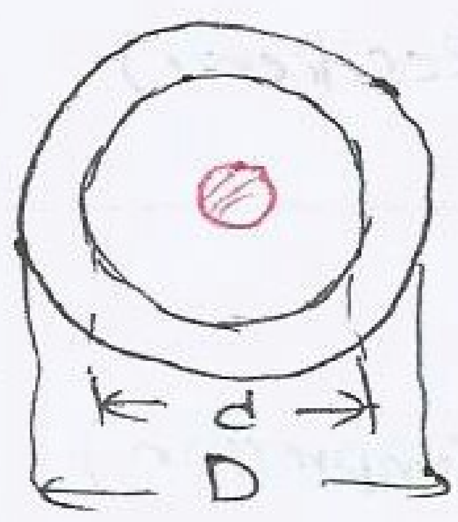
i.e $\boxed{e \leq \frac{2r^2}{d}}$ $\left[\because e = \text{Eccentricity} = \text{~~Distan~~ Permissible distance from Centre} \right.$
$r = \text{Radius of gyration} = \sqrt{\frac{I}{A}}$ & $d = $ ~~least~~ lateral dimension

a) Circular Section



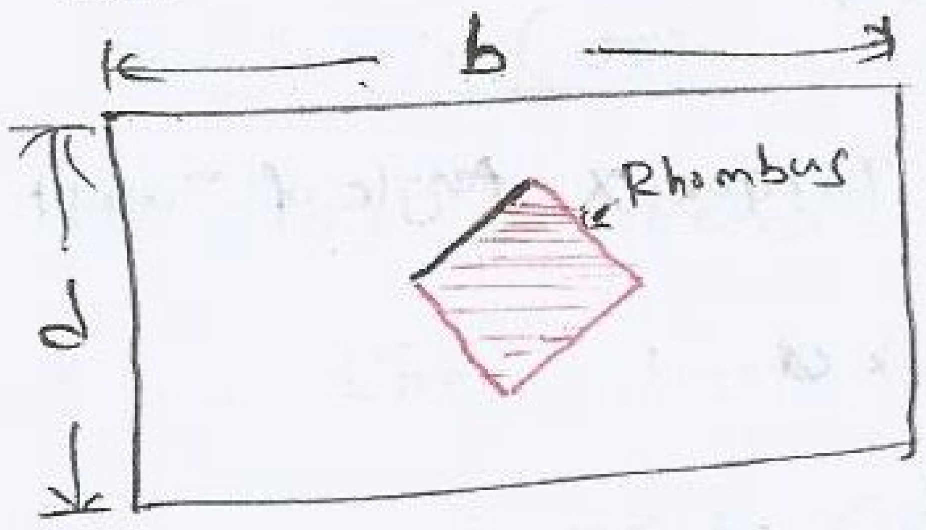
$\boxed{e_{max} = \frac{d}{8}}$

b) Hollow circle



$\boxed{e_{max} = \frac{D^2+d^2}{8D}}$

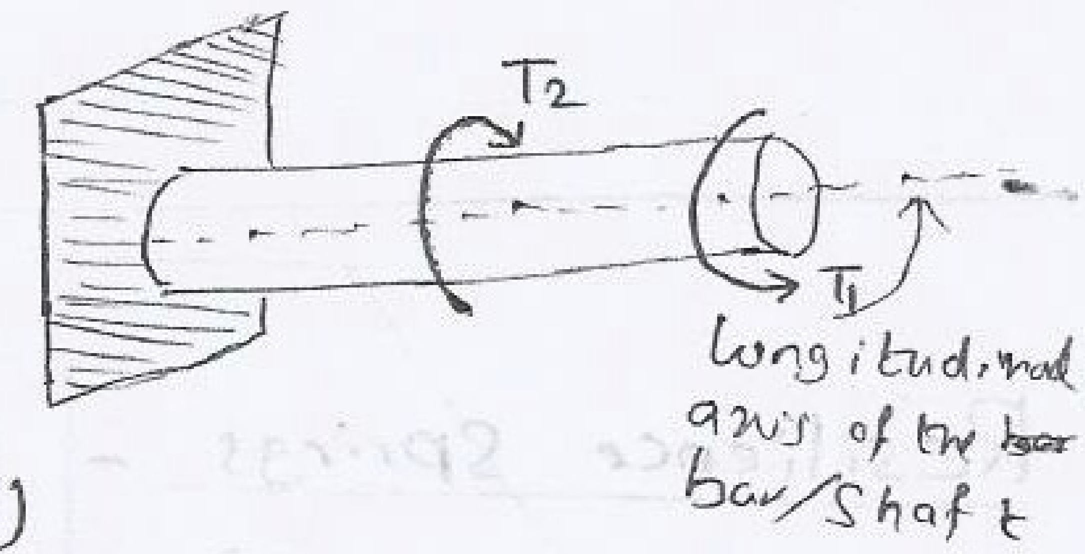
c) Rectangular Section



$\boxed{e_{max} = \frac{b}{6} \text{ (or) } \frac{d}{6}}$

Torsion of Circular Shafts

Torsion refers to the twisting of a straight bar when it is loaded by moments (or Torques) that tend to produce rotation about the longitudinal axis of the bar/shaft.



Torsion equation

$$\frac{T}{J} = \frac{G\theta}{L} = \frac{\tau}{R}$$

T = Avg. Torque applied (kN-m)
J = Polar moment of inertia $= I_{xx} + I_{yy} = \frac{\pi d^2}{32}$
G = Shear modulus or modulus of rigidity
θ = Angle of twist, L = length of shaft, R = Radius $= \frac{D}{2}$
τ = Max. Shear Stress due to Torsion

Notes:

Angle of twist $(\theta) = \frac{TL}{GJ}$

Torsion Rigidity $= GJ$

polar section modulus $(Z_P) = \frac{J}{R} = \frac{\pi D^3}{16}$ (or)

Power transmitted through by shaft

P = Torque (T) x Angular speed (rad/sec) (ω)

$= \frac{2\pi NT}{60}$ watts $\left(\because \omega = \frac{2\pi N}{60}\right)$

To safe against max permissible shear stress/Torque

Dia of shaft $(d) = \left(\frac{16T}{\pi\tau}\right)^{1/3}$

Strain energy due to Torsion

Strain energy stored

$U = \frac{\tau^2}{4G} \times V$ $\left(\because V = \text{volume} = A \times L\right)$

$= \frac{\tau^2}{4G}\left(\frac{D^2+d^2}{D^2}\right) \times V$ ← hollow Shaft

Shafts in Series
$\theta = \theta_1 + \theta_2 + \dots$
Shafts in parallel
$\theta = \theta_1 = \theta_2 = \dots$

x) Combined bending & Torsion

Bending stress $(f) = \frac{M}{Z} = \frac{32M}{\pi d^3}$ & Shear stress $(\tau) = \frac{T}{Z_P} = \frac{16T}{\pi d^3}$

Principal Stresses

$\sigma_{P_1} = \frac{f}{2} + \sqrt{(f/2)^2 + \tau^2} = \frac{16}{\pi d^3}\left(M + \sqrt{M^2+T^2}\right)$

$\sigma_{P_1} = \frac{f}{2} \mp \sqrt{(f/2)^2 + \tau^2} = \frac{16}{\pi d^3}\left(M - \sqrt{M^2+T^2}\right)$

$\boxed{\tau_{max} = \frac{\sigma_{P_1} - \sigma_{P_2}}{2}}$

a) Equivalent Torque $(T_e) = \sqrt{M^2+T^2}$ & b) Equivalent B.M $(M_e) = \frac{1}{2}\left(M + \sqrt{M^2+T^2}\right)$

Resilience

The strain energy stored in a body due to external loading within the Elastic limit.

Proof Resilience - Maximum Resilence upto Elastic limit

Modulus of Resilience - $\dfrac{\text{Proof Resilience}}{\text{volume of material (V)}}$

Strain Energy stored (Resilience) due to	Proof Resilience
1. Bending (i.e Direct tensile/Compression)	$\dfrac{f^2}{2E} \times V$
2. Shear	$\dfrac{\tau^2}{2G} \times V$
3. Torque	$U=$ Avg. Torque $\times$ Angle of Twist $= \dfrac{T}{2} \times \phi$ $= \dfrac{\tau^2}{4G} \times V$ $= \dfrac{\tau^2}{4G} \times \left(\dfrac{D^2+d^2}{D^2}\right) \times V$ ← hollow shaft

Springs

Resilience Springs - Energy stored for future use (eg: Spring in clock).

Carriage (or) Leaf Spring - Absorbs Excess Energy (eg: Spring in bike)

Torsion Spring - A spring primarly subjected to Torsion (or) Twisting.

Closely Coiled helical springs

$n =$ No. of turns
$R =$ Radius of coil $\Rightarrow 8D^3 = 64R^3$
$d =$ dia of spring wire

a) Subjected to External loading (W)

$$\text{Deflection}(\delta) = \frac{8nWD^3}{Gd^4} \quad \& \quad \text{Stiffness} = \frac{\text{load}(W)}{\text{Deflection}(\delta)}$$

b) Subjected to External ~~Torque~~ Couple (M)

$$\text{Total twist }(\phi) = \frac{(2\pi n)MR}{EI} \quad \& \quad \text{Stiffness} = \frac{\text{Couple }(M)}{\cancel{\text{Deflection}}\ \text{Twist}(\phi)}$$

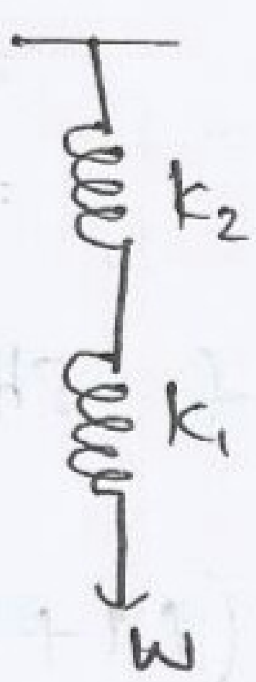
Springs in Series & Parallel

a) Series $(\delta = \delta_1 + \delta_2)$

$$\frac{W}{k} = \frac{W}{k_1} + \frac{W}{k_2}$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

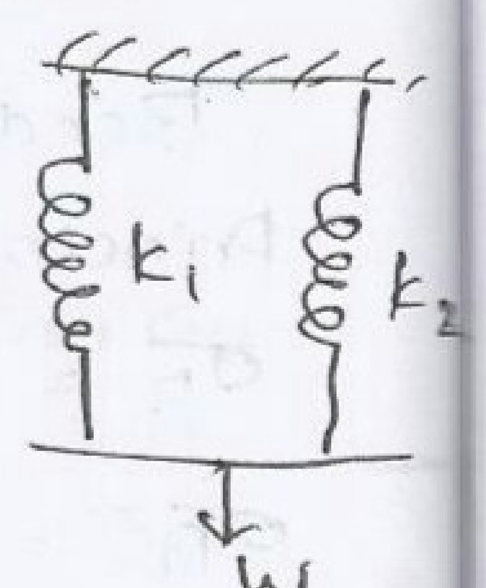
$$\boxed{k = \frac{k_1 k_2}{k_1 + k_2}}$$



b) Parallel $(W = W_1 + W_2)$

$$\delta k = \delta k_1 + \delta k_2$$

$$\boxed{k = k_1 + k_2}$$

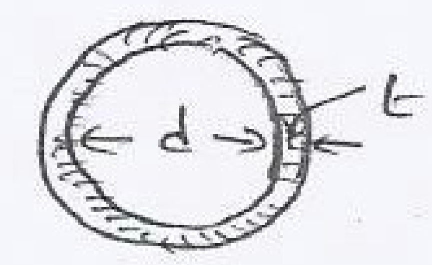


Thin & Thick Cylinders/Tubes

$t < \frac{d}{20}$ (Thin cylinders)

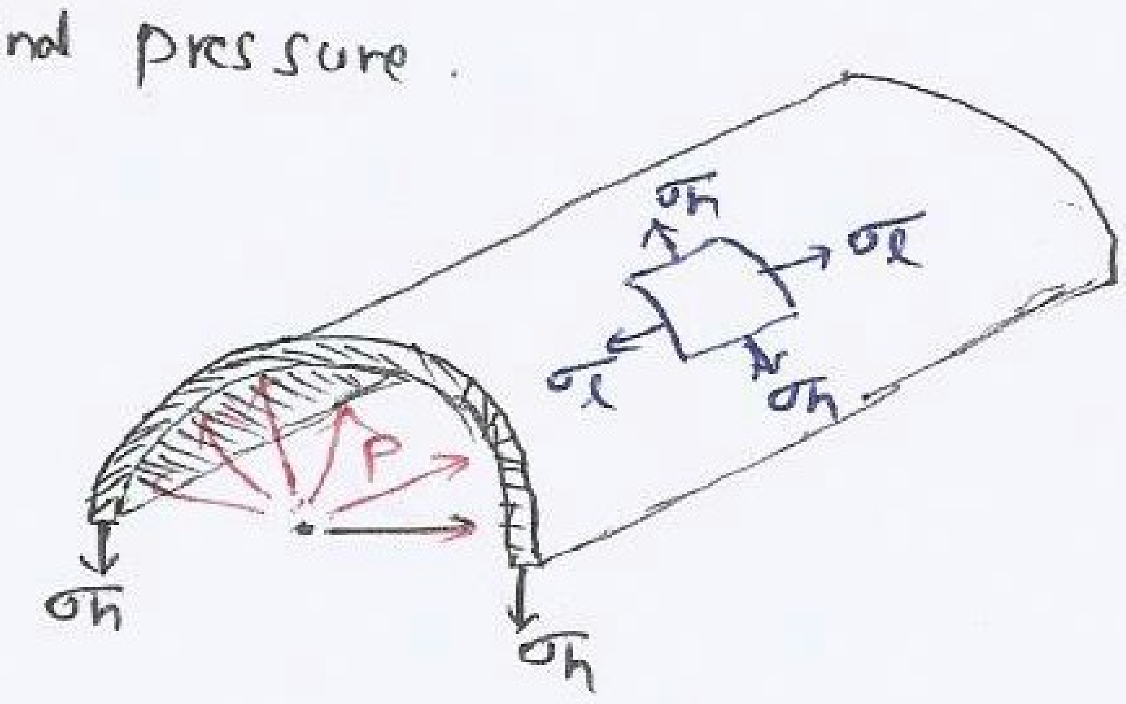
$t > \frac{d}{20}$ (Thick cylinders)

[t = Thickness of the tube
d = Internal dia of tube
P = Internal pressure.



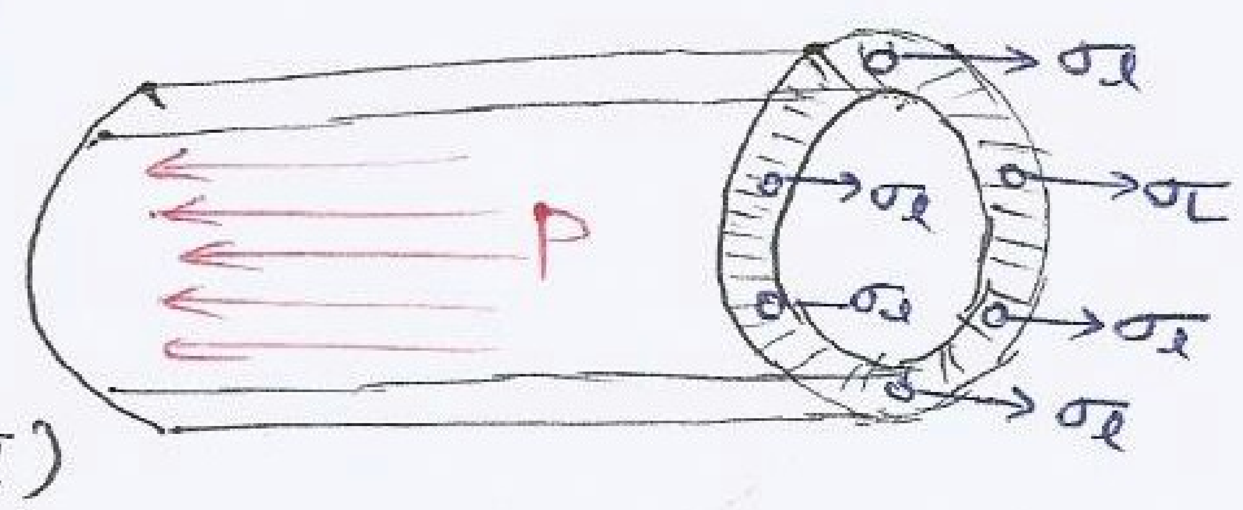
a) Hoop (or) Circumferential stress (σ_h)

$$\boxed{\sigma_h = \frac{Pd}{2t}}$$



b) Longitudinal stress (σ_l)

$$\boxed{\sigma_l = \frac{Pd}{4t}}$$



∴ Hoop stress (σ_h) = 2x Longitudinal stress(σ_l)

→ σ_h & σ_l are tensile stresses.

→ Max Shear stress (τ_{max}) $= \frac{\sigma_h - \sigma_l}{2} = \frac{Pd}{8t}$

*) Efficiency of joints (η)

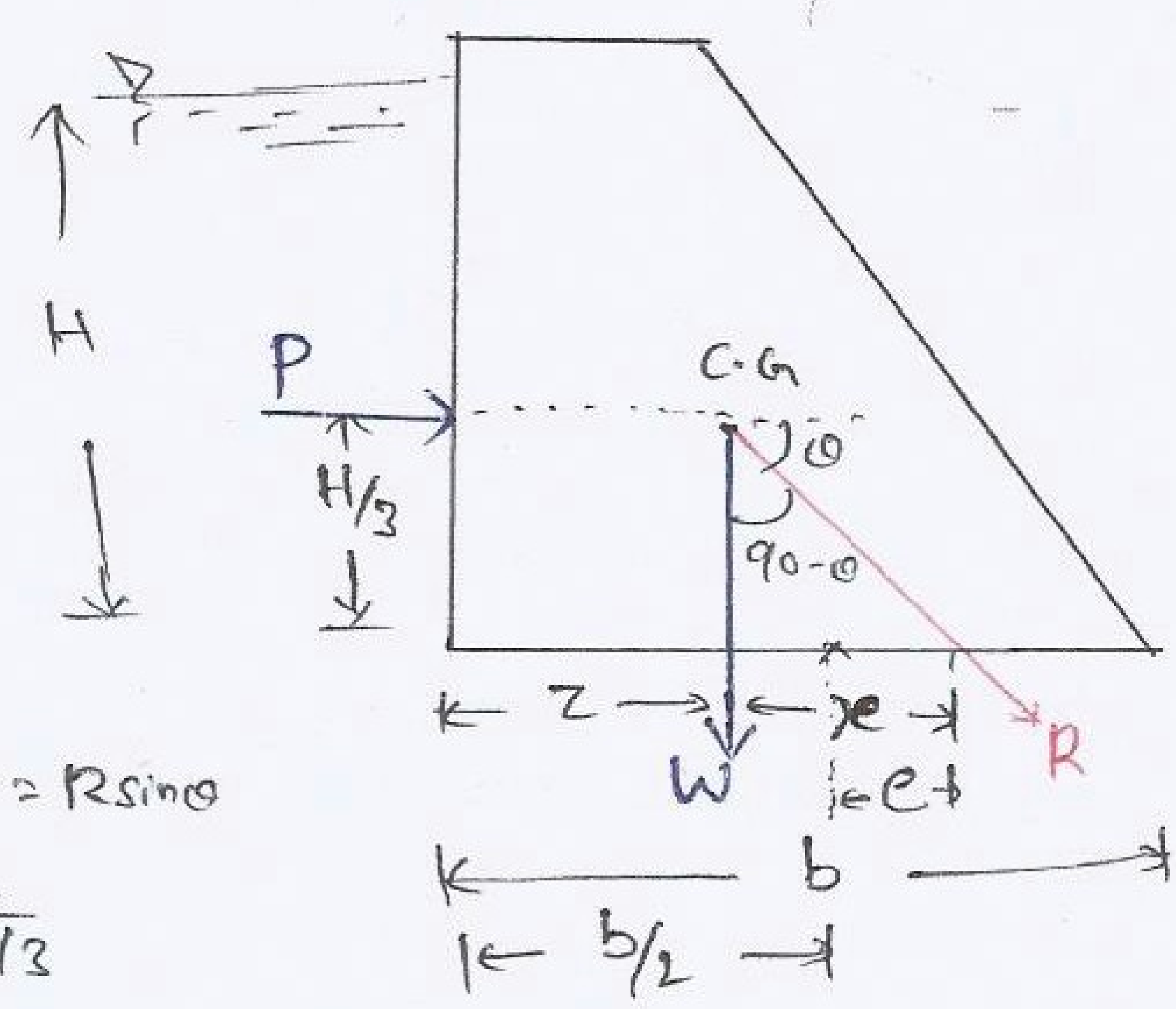
$\sigma_h' = \frac{\sigma_h}{\eta}$ & $\sigma_l' = \frac{\sigma_l}{\eta}$

Dams

Weight of the dam (W) = $\gamma \times (A \times 1)$

Hoz water pressure (P) $= \frac{1}{2}(wH)H$

$= \frac{1}{2} wH^2$



Horizontal distance b/n C.G of the dam to the point where resultant cut the base

$$\boxed{x = \frac{h}{3} \times \frac{P}{W}}$$

[∵ $P = R\cos\theta$ & $W = R\sin\theta$

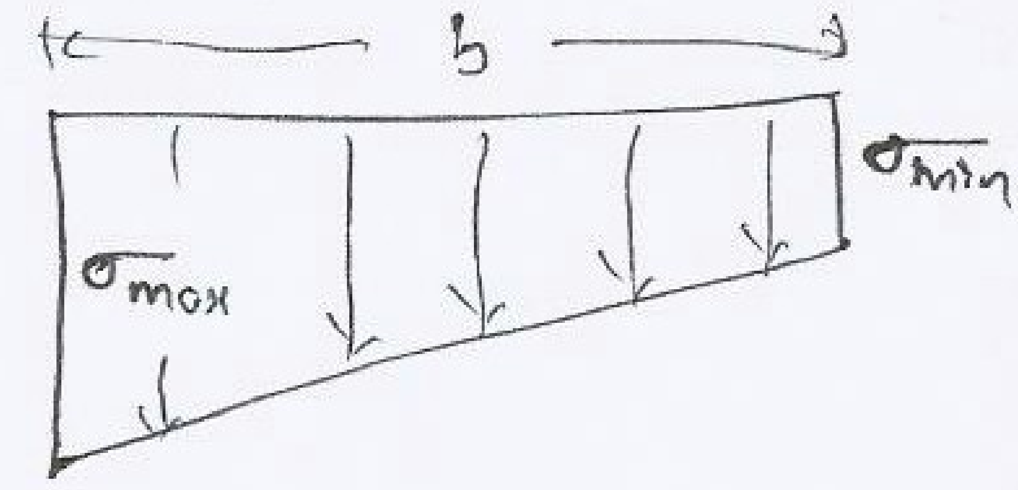
& $\cot\theta = \frac{x}{h/3}$

$\Rightarrow \frac{x}{h/3} = \frac{P}{W}$

Z = Distance from toe - C.G

e = Distance from (mid of base - Resultant cut the base) $= (z+x) - b/2$

∴ $\boxed{e = (z+x) - b/2}$



*) Stresses developed at the base of the dam

$\sigma_{max} = \frac{W}{b}\left(1 + \frac{6e}{b}\right)$

$\sigma_{min} = \frac{W}{b}\left(1 - \frac{6e}{b}\right)$

Note:

If σ_{min} will (-ve) it results ~~overturning~~ Crippling of dam (Due to tensile stresses)

→ Eccentricity (e) < $\frac{(b/2)}{3}$ Middle - Third of the base. (To Resist Tensile stress)

→ Eccentricity (e) < b/2 to resist overturning of dam (∵ R should not cross base)